Time Varying Moments, Regime Switch, and Crisis Warning: The Birth-Death Process with Changing Transition Probability

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Abstract

The sub-prime crisis in the U.S. reveals the limitation of diversification strategy based on mean-variance analysis. A regime switch and a turning point can be observed using a high moment representation and time-dependent transition probability. Up-down price movements are induced by interactions among agents, which can be described by the birth-death (BD) process. Financial instability is visible by dramatically increasing 3rd to 5th moments one-quarter before and during the crisis. The sudden rising high moments provide effective warning signals of a regime-switch or a coming crisis. The critical condition of a market breakdown can be identified from nonlinear stochastic dynamics. The master equation approach of population dynamics provides a unified

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theory of a calm and turbulent market.

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1. Introduction

Current economic literature has no consensus on the pertinent representation of financial crises. A neo-classical perspective, such as the Diamond-Dybvig model [1] and the noise trader model [2], only give qualitative descriptions of multiple equilibriums, but they do not offer any operational indicator in defining different regimes in terms of empirical observation. The evolutionary perspective is interested in time patterns of historical events. Minsky and Kindleberger made a stylized description of three types of crises [3]. They made a verbal description of duration and phases in historical records, but did not suggest any quantitative measurement of a crisis in terms of a time series. These two perspectives are intuitive in theoretical ideas but impractical in quantitative analysis. How to diagnose a coming crisis from empirical data is an open issue both in theory and practice.

To bridge the gap between qualitative theory in a historical perspective and quantitative measurement in numerical experiments, two quantitative approaches are used for diagnosing a crisis in a time series analysis.

The first approach tries to identify some thresholds for market bubbles from an excessive volatility of market indicators [4]. However, the following experiment demonstrates that price level changes are not a reliable indicator for a crisis. Based on historical records, we may rank a one-day price drop from high to low. The largest one-day drop of 22.61% occurred on Oct. 16, 1987, which did not develop into a significant crisis. The 2nd largest one-day price drop was 13.47% on Oct. 25, 1929. Among the top 30 events with one-day price drops larger than 6.54%, 17 events (57% of the top 30 events) occurred during the Great Depression. 4 events (13%) happened during the 2008 Crisis. 9 events (30% of total observations) did not trigger a full-fledged crisis. Their price drops ranged from 6.54% on Sept. 23, 1955 to 22.61% on Oct. 16, 1987. Clearly, judging a crisis based on the magnitude of level changes would be quite subjective, because there is no theory revealing the relationship between crises and price change magnitudes.

The second approach is based on some ad hoc static models, such as a fat tail distribution or a log-periodic model. Its strength is its mathematical simplicity, but its weakness is that it is hard to put its findings in a historical perspective. It is widely believed that high-frequency financial data have the accumulated probability distribution function (pdf), which decays with an inverse cube [5]. Preis and Stanley *et al* [6-7] found that the trading volume will turn large when the market trend switches, but a trend switch is not sufficient for a full fledged crisis. Power law provides little information for timing a crisis since its data requirement implies a large time window in statistical analysis. Critical information on a crisis is not reliable from power law, since the tails could decay faster than power law. Although we may have better approximations of the sample distributions by means of other models, we still lack useful information on crisis warnings [8]. Methodologically speaking, a stable pdf in a long time window cannot offer a real-time monitor of the degree of market stability. For managing a financial crisis, we need an effective indicator in a short time window. Sornette [9] noticed that some large market crashes are outliers of a stable market (such as the crashes in Apr. 2000 and Oct. 1987). He identified a log-periodic pattern from a possible bubble buildup process. His problem is that he needs a theory to justify his model, since there is little evidence of harmonic waves from business cycle data. We found solid evidences of continuous-time color chaos with an erratic amplitude but a narrow frequency band from macro and stock indexes, which are nonlinear and aperiodic in nature [10, 11].

We developed a third approach, which reveals the degree of market instability and the timing of a coming crisis from a non-stationary time series analysis. Our numerical representation is high moments in statistics. Our theoretical framework is the master equation in statistical mechanics. Our simplified model for market price up-down movements is the population model of the birth-death process. The Master equation has been used in option pricing and herd behavior in the financial market [12, 13]. Schrödinger's Principle of Large Numbers sheds light on market resilience from macro and finance data [14, 15]. The birth-death process is the simplest population model, which ensures the Principle of Large numbers in stochastic dynamics [14]. Market instability and crises can be described by a regime switch in the nonlinear stochastic process. Our numerical experiments show that the high moment (3^{rd} to 5^{th} moment) returns from stock market indexes can be a useful indicator of economic complexity [16] and market instability. Non-linearity and complexity in economics and physics have many interesting features, such as power law, fractal, network, and criticality [5-7, 17-21]. Understanding economic complexity will open new ways of research in financial economics.

This work shows that high moments reveal critical information on dynamical instability and crisis timing from a non-stationary financial time series. It is known that business cycles are non-stationary in nature. The 1st and 2nd moment reveal limited information of a changing market regime in volatility and returns [22]. Markowitz [23] realized that the second order moment is only feasible under equilibrium arbitrage and high moments may be the cause of market speculation, but he discarded higher than two moments because he could not figure out the economic interpretation for high moments at that time. We found out that the dramatic rising of higher than 2nd moments (especially the third to fifth moments) provides a clear indicator of a dynamic regime switch and a likely coming crisis. Our high moments approach has one advantage: conceptually, a statistical moment is clearly defined both in statistics and statistical mechanics, so that we can develop a unified approach in empirical analysis and theoretical modeling. In practice, there is an operational problem in calculating the moments. As we know, the mean, the cumulant, and probability distribution are stable only in controlled experiments in physics labs. Economic time series are not obtained from controlled experiments and they are non-stationary in nature. Therefore, numerical moments can only be calculated through a moving time window. Thus, the working definition of moments is an empirical issue in financial analysis. Its usefulness should be verified by its power in explaining observed patterns and historical events. We tested high moments measured by a short time-window of one period that serves as an indicator in studying critical phenomena in economics. We found that the dramatic rise (1000 times or more) of high (3rd to 5th) moments before and during a crisis, which may serve as the signal of a market breakdown [24]. Now we have a better understanding of why diversification strategy failed during the sub-prime crisis. The mean-variance approach ignores excess speculation driven by high moment deviations, which is significant before and during a crisis period. For diagnosing a crisis generated by herd behavior, we introduce a population dynamic of the birth-death process for describing up-down price movements by means of a stochastic differential equation. The Black-Scholes option pricing model based on the representative agent model of geometric Brownian motion can be extended to the more generalized population model of the birth-death process, we would discuss this issue elsewhere [25]. In this article, we will focus on diagnosing a financial crisis by means of high moments representation and identifying a crisis condition from the birth-death process, since social interaction in collective action is the source of herd behaviors and market fads in behavioral economics [26].

In the following sections, the limits of a stable distribution and the advantages of high-moments representation are discussed in section 2. The up-down price dynamics described by the birth-death process and the high moment representation of its solution is introduced in section 3. The critical point of a financial crisis is given as the existence condition of the solution equation. Empirical observations of changing high moments before and during a crisis are shown in section 4. The nonlinear and non-stationary nature of financial dynamics is concluded in section 5.

2. The theoretical framework of high-moment representation in the Fokker - Planck equation

We first discuss the limits of a static distribution, before introducing the theoretical framework of high moments and the Fokker-Planck equation. Our choice is not for mathematical elegance, but for practical analysis of a financial crisis.

2.1 The limits of static distribution: the case of accumulated probability distribution function (pdf) from daily data

Space and time scale plays a central role in physics issues. Classical mechanics and quantum mechanics operate in different space scales. There is a similar issue in time or frequency scales in economics and finance. Policy issues are studied in frequency scales using daily to annual data. Recent interest in high frequency data in minutes is used for analyzing power law and trading psychology. We use daily data for crisis analysis since historical events are recorded in days, not in minutes.

We should point out that a fat tail distribution and power law with identical probability provides little information on structural change caused by liberalization policy in past three decades in the U.S. So far as we know, the policy implications of the power-law are not clear. You may demand tight regulation or argue for the difficulty in regulation under the situation of power law. Static distribution in financial analysis shows an interesting pattern of non-linearity in financial dynamics, but it is not capable of identifying the timing of crises because a regime switch can only be observed from a non-stationary process. In a static distribution, a large deviation may occur at an identical probability in any time, while the recorded economic crisis may not be evenly distributed. More likely, crises occur in clusters or during structural changes. A sudden change in economic activities may be similar to a phase transition in physics. We examine the case of an accumulated probability distribution function (pdf), which is widely used in statistics but rarely applied in crisis analysis.

We find that the stretched exponential (SE) pdf can fit the common-frequency (daily) data of the Dow-Jones Industry index at all quantile ranges. As shown in Fig.1, the SE fits daily data very well. There is an explicit breakpoint in the negative tail. The outliers beyond the breakpoint are large deviations during financial crises, including several events from 1929 to 2008 as we mentioned before.

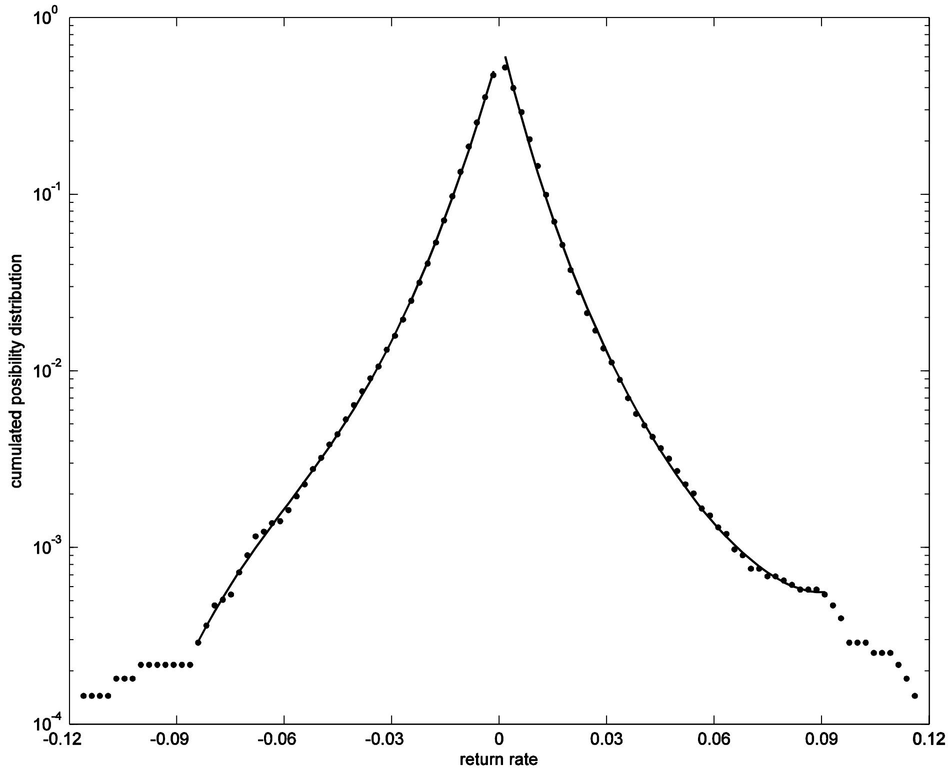

FIG. 1. The cumulated probability distribution of the Dow-Jones industrial index (dji). Its daily change rate ($r = x_{t+1} / x_t - 1$) is from 2-Jan-1900 to 1-Sep-2010 (dotted line), and its SE fitting was by means of the 4th degree polynomial (solid line). Total data has 27724 points.

In FIG. 1, for the negative part, the cumulated pdf is $P(r) = \exp(-5.959\times10^4 r^4 - 78.33r^3 + 1249r^2 + 161.8r - 0.4371)$, with adjusted $R^2 = 0.9993$. For the positive part, the cumulated pdf is $P(r) = \exp(8.664\times10^4 r^4 - 1.925\times10^4 r^3 + 2258r^2 - 191.5r - 0.1977)$, with adjusted $R^2 = 0.9993$. Where r is the daily changing rate of dji. There is an explicit breakpoint in the negative tail at $r = -0.084$.

In FIG. 1, we can see three features of a static probability distribution. First, historical data do not follow a homogenous probability distribution. Many outliers cannot fit with a smooth curve implied by a static probability distribution. There is no clue to different market regimes, say, a stable regime and a crisis regime. Second, extreme deviations rarely occur. Numerically, it is not reliable to find the breakpoint from a stable distribution. Third, there is no information on the timing of large market deviations, since scattered points in pdf do not show their occurrence in clusters or as evenly distributed. In contrast, these problems can be solved, if the distribution function is not static, but time varying. Based on a dynamic approach, we can explain both the observed non-linear SE fitting curve and the break point in the tails.

2.2 Economic implications of high moment representation under non-equilibrium framework

High-moment representation sounds trivial in non-equilibrium physics but revolutionary in equilibrium economics since neoclassical economics often excludes the existence of nonlinearity and multiple equilibriums in the theoretical formulation of dynamic general equilibrium models.

In mathematical physics, a probability distribution can be described by a series of moment representations to infinite order. In financial practice, standard mathematical models in asset pricing are mainly confined to mean and variance. The orthodox belief in an efficient market simply implies the non-existence of instability and crisis in a financial market. To our knowledge, few economists have studied the relation between high moments and financial instability. The importance of high moments is visible only when we change our theoretical perspective from a stable Gaussian distribution to a non-stationary time-varying non-Gaussian distribution. This is a paradigm change that is similar to a change from a geocentric circle model to a heliocentric ellipse model in planet motion. Levy distribution and power-law were found using financial data by Mandelbrot and Stanley [27, 28]. So far as we know, there is little link between a static Non-Gaussian distribution and non-stationary financial dynamics. Based on our observations, crisis occurred as clusters of correlated large deviations. Therefore, our general framework of a non-stationary time-varying distribution with changing high moments is capable of identifying a crisis in a specific time period. From empirical analysis, we can clearly distinguish a calm from a turbulent period, which provides strong evidence of a phase transition in a financial market.

The standard stochastic model in finance is the random walk or geometric Brownian motion. The Black-Scholes model in option pricing can be derived from the master equation [29]. But this work did not reveal a new economic mechanism in the financial market. The Ising model and a Gaussian-type (Maxwell) distribution have been introduced in public opinion and economic geography [30, 31]. However, there are two problems in applying equilibrium statistical mechanics to economic dynamics. First, the Maxwell distribution in statistical mechanics has no theoretical foundation in economics, since social temperature is not defined in economics. Second, an empirical financial time series shows little evidence of a stable Gaussian distribution. We need new ideas from non-equilibrium statistical mechanics and nonlinear stochastic dynamics.

From our study, the intensity of social interaction characterizes a different social atmosphere in collective action; changing the distribution from a unimodal to a bimodal distribution may indicate a phase transition in economic dynamics [10]. We consider the birth-death process in physics and chemistry as a better alternative to geometric Brownian motion in financial economics for two reasons. First, the geometric Brownian motion model only has one agent but the birth-death process has N agents. By introducing population dynamics, the nonlinear up-down price movements may be associated with herd behavior, which cannot be explained by the representative model of geometric Brownian motion. Second, the birth-death process is the simplest as well as a powerful approach, which could construct a unified framework, not just for finance theory, but also for micro industrial theory and macro business cycle theory. For examples, we could explain why Schrödinger's Principle of Large Numbers is valid for macro and finance analysis; we could explain abnormal movements in option pricing [25]; we can reproduce all the existing financial models from a birth-death process as Cox and Ross have shown in 1976 by keeping the detailed balance [32]. To our knowledge, no other models can provide a simpler and wider explanation than the birth-death process in finance and macroeconomics.

Theoretically speaking, crisis dynamics may radically differ from normal financial dynamics. Therefore, we do not claim the FK equation as the ultimate formulation for crisis dynamics. We only use high moments and the Fokker-Planck equation of a birth-death process as a diagnostic tool in studies of time series including the crisis period. We found two different dynamic regimes. In the second regime, we found a non-stationary distribution and a non-linear mechanism that are observable in the high-moment representation of a non-linear birth-death process. These observations reveal possible dynamics for understanding financial crises.

3. The master equation of the birth-death stochastic process and the high moment representation in empirical analysis

Non-linearity can generate stability or instability with varying intensity. If the non-linearity is weak, market resilience will follow Schrödinger's rule of life as we observed business cycles (their periods are varying between one and ten years) in a long time window with low frequency (monthly or quarterly) data [11]. And if the non-linearity is high, the market may enter a crisis regime in a short time window using daily data. We will analyze daily data in this article.

3.1 The dynamic equation for time-varying distribution and the high-moments representation in the Fokker-Planck equation

In statistical mechanics, the evolution of a time-varying distribution and its high moment representation is governed by the master equation. For the financial market, the simplest master equation is the birth-death process (BD), which is the proper model for persistent fluctuations in the financial market [11, 24]. Non-linear BD implies a nonlinear feedback mechanism, which can be explained by high moments representation. We may classify market movements into two dynamic regimes: A calm market is characterized by finite orders of high moments in addition to mean (the growing trend) and variance. A turbulent market is characterized by rising high moments that are several orders larger than the variance. Theoretically speaking, we may not have a statistical description of the system during the realm of panic crises, but we can use the high-moments representation as a tool to observe an ongoing crisis.

BD can be described by the following master equation:

$$\frac{\partial P(X,t)}{\partial t} = W_{+}(X-1)P(X-1,t) + W_{-}(X+1)P(X+1,t) - [W_{+}(X) + W_{-}(X)]P(X,t), \quad (1)$$

Unknown
Field Code

where $W_{+}(X) = W(X+1 \mid X)$, $W_{-}(X) = W(X-1 \mid X)$ are birth and death rates, respectively.

In our case, the birth rate means the probability of the price moving up one price unit, and the death rate is the probability of the price moving down one price unit. Both probability functions are nonlinear in state and time. It implies a third possibility of non-change in price. In comparison, the random walk model in finance only considers a constant possibility of up or down movements in finance.

The master equation approach is widely used in statistical mechanics in both physics and chemistry, which in economics, is also called social dynamics in studies of interacting agents [18, 30, 31, 33]. Its application depends on its formulation of structure for specific dynamics. For example, transition probability in social dynamics assumes the Gaussian form; while our Non-Gaussian transition probability is estimated from empirical data.

The stationary solution to the master equation is straightforward at the detailed balance condition $W_{+}(X-1) = W_{-}(X)$ [34].

We can directly estimate a transition probability W from a financial time series. We can define the minimal counted financial index change (0.01 point) as unit 1 in price movement. Its stationary solution is the following:

$$P^{st}(x) = e^{-\phi(x)} \tag{2}$$

Where $\phi(x) = \int_0^x \ln\left[\frac{W_-(v)}{W_+(v)}\right] dv$, and $X = 100x$ in Eq.(2).

Eq. (2) can be expressed by means of the nonlinear SE with a Taylor expansion of $\phi(x)$. In this paper, the master equation is useful for calculating the breakpoint between the stationary pdf and turmoil crisis.

In terms of the Poisson representation of the Fokker-Planck equation [35], the nonlinear BD can be approximated in a calm market as

$$\begin{aligned}
\frac{\partial F}{\partial t} = &-\frac{\partial}{\partial \alpha}[(b_1 - d_1)\alpha + (b_2 - d_2)\alpha^2 + (b_3 - d_3)\alpha^3 + (b_4 - d_4)\alpha^4]F(\alpha, t) \\
&+\frac{\partial^2}{\partial \alpha^2}[b_1\alpha + (2b_2 - d_2)\alpha^2 + (3b_3 - 2d_3)\alpha^3 + (4b_4 - 3d_4)\alpha^4]F(\alpha, t) \\
&-\frac{\partial^3}{\partial \alpha^3}[b_2\alpha^2 + (3b_3 - d_3)\alpha^3 + (6b_4 - 3d_4)\alpha^4]F(\alpha, t) \\
&+\frac{\partial^4}{\partial \alpha^4}[b_3\alpha^3 + (4b_4 - d_4)\alpha^4]F(\alpha, t) \\
&-\frac{\partial^5}{\partial \alpha^5} b_4\alpha^4 F(\alpha, t).
\end{aligned} \tag{3}$$

Where we expand transition probabilities as $W_+ = b_0 + b_1 f_1 + b_2 f_2 + b_3 f_3 + b_4 f_4$, $W_- = d_0 + d_1 f_1 + d_2 f_2 + d_3 f_3 + d_4 f_4$, with $f_0 = 1$, $f_1 = x$, $f_2 = x(x-1)$, $f_3 = x(x-1)(x-2)$, and $f_4 = x(x-1)(x-2)(x-3)$.

We only keep 5 orders of high moments; because a 4th order polynomial is good enough for describing $W_\pm$ in the real market [36]. $F(\alpha, t)$ is the Poisson series where $P(x,t) = \int d\alpha \frac{e^{-\alpha}\alpha^x}{x!} F(\alpha, t)$.

3.2 The critical condition for crises in high moment representation

Equilibrium theory mainly considers the first (mean value) and second moment (variance), non-equilibrium models study social behavior with higher moments [16]. Eq. (3) describes a non-linear system with a time-varying distribution, which may have critical phenomena. In its turmoil regime, large deviations will be densely clustered, and all moments will diverge [35]. The critical point in Eq. (3) is

$$(b_1 - d_1) + 2(b_2 - d_2)x + 3(b_3 - d_3)x^2 + 4(b_4 - d_4)x^3 = 0. \quad (4)$$

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Eq. (4) has three implications. First, only nonlinear dynamics can generate critical phenomena. The linear case only has the first two terms in Eq. (3). Second, Eq. (4) indicates the condition of diverging high moments, since the left side of Eq. (4) would appear in the denominator when solving Eq. (3) using the perturbation method [35]. Third, Eq. (4) occurs at the changing point where a market trend may turn from optimism into pessimism, so that a market panic or bubble burst could happen at this turning point.

We may test our theory by historical events such as the 2008 financial crisis in another paper [36]. Therefore, we do have valuable information on a crisis warning by constantly monitoring high moments within a moving time window.

3.3 Calculating high moments in a non-stationary time series

In theory, both equilibrium statistical mechanics and mathematical economics define the cumulant or moment in terms of deviations to the mean [23]. The noise or deviation term is: $\varepsilon_t = x_t - E(x_t)$.

When analyzing empirical data, econometricians soon realized that $E(x_t)$ varies with the changing width of time windows. To simplify empirical analysis, econometricians substituted $E(x_t)$ with x_{t-1} which became a standard form in financial economics. In a stochastic process, it implies that the realized prices are martingales. It is the mathematical simplicity, not the empirical relevance, which serves as the foundation of the so-called efficient market hypothesis (EMH) [37].

Our framework adopts the moment concept in two ways. In the previous case of asset pricing, we define $E(x_t)$ as a moving trend in a long time window, which is a local approximation of the cumulant. In the current case of crisis diagnosis, we simply adopt the shortest time window within one period. This is the same practice in econometrics when financial models do not assure the existence of a smooth or stable expectation.

In numerical calculation, we define the *k*th un-annualized moment as $\frac{\sum_{i=1}^{N}(x_i - E(x))^k}{N}$, where $E(x)$ is the mean of x.

To observe high moments of a non-stationary financial time series, we take

$$\frac{\sum_{i=1}^{N}(x_i - E(x_i))^k}{N} = \frac{1}{2}\left(\frac{\sum_{i=2}^{N}(x_i - x_{i-1} - \mu_{i-1})^k}{N-1}\right), \quad (5)$$

Unknown Field Code

where μ_t is the expected growth rate.

We use $\frac{\sum_{i=2}^{N}(x_i - x_{i-1})^k}{N-1}$ to approximate the *k*th un-annualized moment in section 4, because the daily growth μ is small.

We will experiment by calculating moments with $\varepsilon_t = x_t - x_{t-1}$. Our result will show that our calculation of high moments WOULD reveal new features in characterizing calm and turbulent regimes in a financial market. A calm regime can be considered as the linear approximation (or EMH) of the birth-death process when $t \to 0$.

4. Empirical observations of changing high moments and regime diagnosing in financial markets

In our study of financial crises, we choose a quarterly time window in analyzing daily time series. The length of the time window series varies with available data for a specific issue in research.

The original financial time series with a growing trend and erratic fluctuations are hard to distinguish a turbulent market from a calm market. High moment representation provides a clear picture of a regime switch between calm and turbulent markets. We apply high moment analysis to two widely used financial indicators: the Dow-Jones industry daily index, and TED, which is the interest rate spread between a three-month Eurodollar LIBOR rate and a 3 month U.S. Treasury Bill rate. In calculating the quarterly moments of the Dow-Jones industry daily index and TED spread, each quarter contains approximately 61 trading days, so that N=61. We will see that high moment representation is capable of distinguishing a turbulent regime from a calm regime. Dramatically rising high moments before and during a crisis could serve as an effective warning signal of a coming crisis.

4.1 Complex patterns of high moments from the Stock Index with or without financial crises

In FIG. 2, we compare two representations of the Dow Jones Industrial Daily Index from 2-Jan-1900 to 1-Sep-2010. The original time trajectory (the dashed line) is the natural logarithmic daily close price series. Its growing trend with erratic fluctuations is the common feature of many macro and financial indexes. Its non-stationary feature is rarely seen from physics data in controlled experiments. The moment representation (the solid line) is calculated through a quarterly moving time window. You may see big contrast between large and small peaks from the moment representation. The large peaks reveal valuable information on market instability and possible crisis.

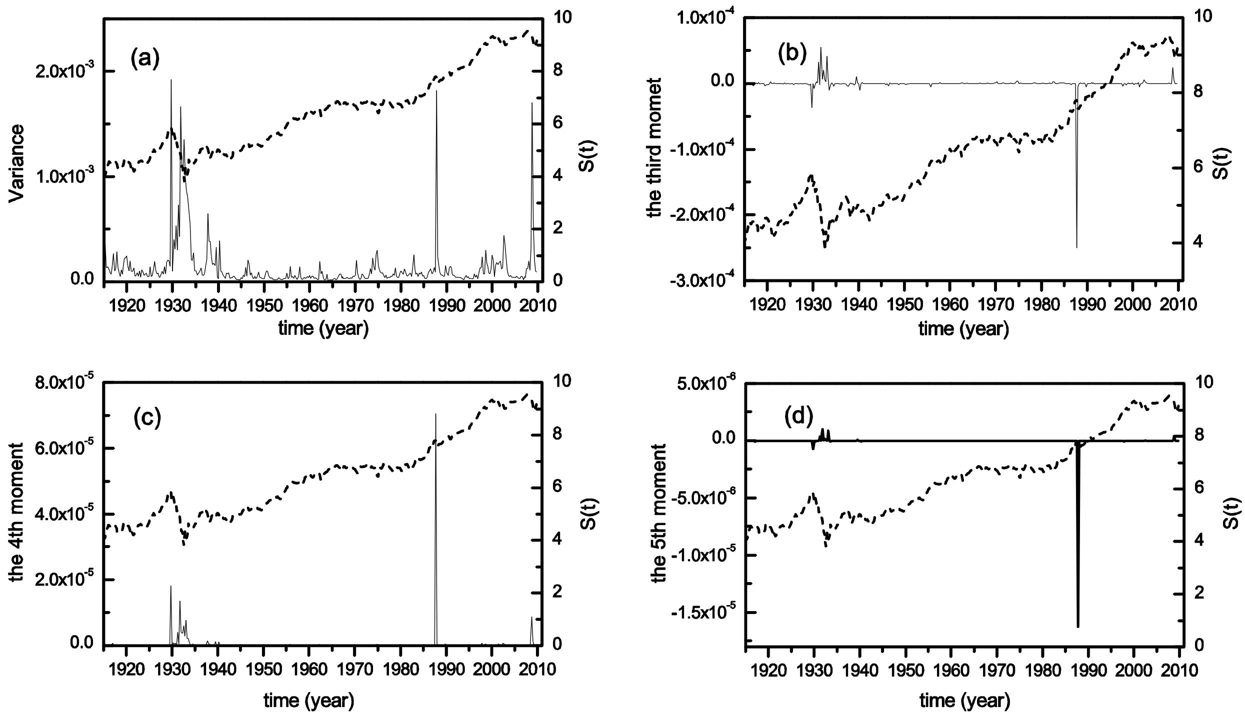

FIG. 2. The quarterly moments (solid lines) of the Dow-Jones Industrial Average (DJI) index. The original S(t) (dashed lines) is the natural logarithmic daily close price series. Each point in the solid line is calculated with a moving time window; its width is one quarter. Plots (a), (b), (c) and (d) correspond to $2^{nd,}$ 3^{rd}, 4^{th} and 5^{th} moment, respectively. The magnitudes of each moment representation are 10^{-5} for variance, 10^{-8} for 3^{rd} moment, 10^{-9}

for 4^{th} moment, and 10^{-11} for 5^{th} moment. The daily data were from 2-Jan-1900 to 1-Sep-2010 with 27724 data points.

Here we choose $\sigma_0^2 \sim 10^{-5}$ as the normal level. We would consider high moments when they reach the level of $10^{-1}\sigma_0^2$ or higher.

Time-varying high moment representation reveals more information than a time series model generated from a static distribution. In FIG. 2, we can see that sharp peaks appear during turbulent periods in the financial market. Four dramatic peaks can be identified from their corresponding periods: the quarters of 4^{th}/1929, 4^{th}/1931-4^{th}1933, 4^{th}/1987, and 4^{th}/2008. All the timing of major peaks is consistent with records of a historical crisis. Certainly, there are small peaks in addition to major market fluctuations, which are less useful for crisis diagnosis at this stage.

Five patterns can be observed from FIG. 2.

First, the magnitude measurement can be classified into two different dynamic regimes: a calm market and a turbulent market. For a calm market, the magnitude of high moments (3^{rd} to 5^{th} moment) is quite small (say, less than 0.1% to 0.001% of the magnitude of the variance) compared to the 2^{nd} moment during the periods of a calm market. This observation shows that the mean-variance model in neoclassical finance theory is a good approximation only for a calm market when the higher than 2^{nd} moments can be ignored [23]. For a turbulent market, the magnitudes of the higher moments typically increase 100 to 1000 times, which occurred in the quarters before and during the crisis period, so that the magnitudes of high moments are comparable to or even larger than the usual magnitude of variance. This observation is true for 3rd/1914, 4th/1929, 4th/1931-4th1933, 4th/1987, 4th/2008. There are less sharp peaks at 1st/1907, 3rd/1939, 2nd/1940 and some small peaks around 2000. Therefore, mean-variance analysis and the Black-Scholes option pricing model [38] would breakdown one-quarter before and during a market turmoil, since the variance is not constant and high moments cannot be ignored in financial analysis. That is why a portfolio diversification strategy and a derivative market could fail during a crisis. This observation is beyond the scope of linear models of financial theory.

Second, a turbulent market is only a necessary but not a sufficient condition for a market crisis. The magnitudes of higher moments alone cannot tell the difference between a temporary market panic (such as the Oil Price Shock in 1973 and the Stock Market Crash in 1987) and a persistent depression (such as the Great Depression in 1929 and the Grand Crisis in 2008). The length of a crisis duration varies greatly from temporary panic to persistent depression. However, our method of a high moments one-quarter warning greatly reduces forecast error in comparison with the level change monitoring discussed in the introduction.

Third, the observed length of a turbulent period depends on our observation time window. For example, the diverging period of the Great Depression was 4th/1929 to 4th/1933 with two dramatic peaks and a short dip in between. By observing the moving average of quarterly data in FIG. 2, The length of the turbulent period near the first peak was only 7 successive trading days; while the length of the turbulent period of the 4th quarter in 2008 was 121 days, based on Table I from daily close data.

Table I. The durations of panic days in financial crisis

	4^{th}/1973	4^{th}/1987	4th/1929	4^{th}/2008
duration (trading days)	0	2	7	121

In Table I, we can see that the oil price shock did not cause a persistent turbulent market. The Stock Market Crash in 1987 only lasted for 2 days and did not turn into a full-fledged financial crisis. The 1929 stock market crash lasted for 7 days and was the symptom of a panic. We may speculate that the 2008 Crisis must have been generated by a deep structural problem, since it lasted for 4 months! The 2008 Crisis in the U.S. turned into a global financial crisis within these four months, during which both high-moments of dji and TED diverged (seen in the following FIG. 3).

Current tools for qualitative diagnosis of a financial crisis are mainly based on the level of deviations from trend or percentage changes [39]. The higher moments of financial indicators provide a new tool in the quantitative diagnosis of financial crises.

4.2 High moment behavior for interest spreads

So far, we mainly observed the high moment behavior for the Dow Jones Index before and during the 2008 crisis. Critical readers need to double check the generality of the high moment in diagnosing market instability. Let us consider a second index to demonstrate a similar pattern of high moments from financial indexes. In financial economics, an interest rate spread TED is widely used as an indicator of a changing market risk [40]. A similar high moment analysis of interest rate spread data is shown in FIG. 3, which confirms the non-stationary nature of a financial crisis. To save space, we only demonstrate the 3rd moment of TED spread below.

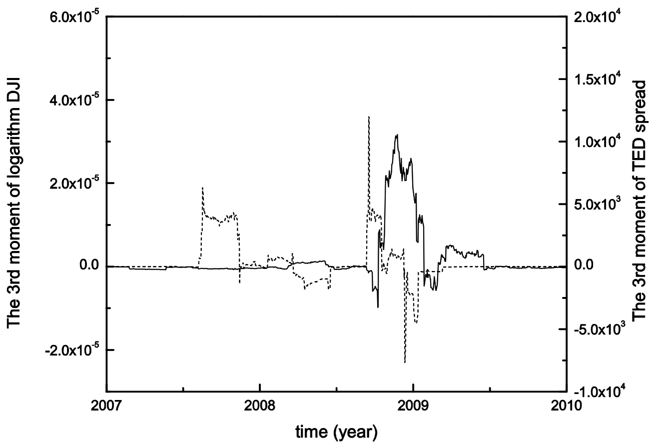

FIG. 3. The 3rd moments of the logarithm DJI (solid line) and TED spread (dashed line) during the 2008 financial crisis. The 2nd, 4th, and 5th moments have similar patterns. TED is the interest rate spread between a three-month Eurodollar LIBOR rate and a 3-month U.S. Treasury Bill rate. Each point is calculated with a moving time window; its width is one quarter. The daily data is from 02-Jan-1990 to 22-Sep-2010 with a total of 5403 points.

From FIG. 3, three observations are of interest when studying financial market dynamics:

First, the interest rate movements in the international money market are closely correlated with the U.S. stock market.

Second, the TED spread is more sensitive than the DJI index in market sentimental, since the TED peak is slightly ahead of DJI peak. This finding may improve our ability to warn of a coming crisis. It is also useful in studying market psychology. A dramatic rise of the interest rate spread in the international money market signals a coming crisis, which is a familiar experience for monetary traders. For the TED spread, all the moments diverged simultaneously on Sep. 12, 2008 and re-appeared on Nov. 28, 2008. For the logarithm DJI, only the 5^{th} moment diverged on Sep. 26, 2008, where other moments grew slowly. The largest magnitude of logarithm DJI's variance appeared on Dec. 05, 2008.

Clearly, arbitrage activity is a double-edged sword, which could generate both negative and positive feedback in market exchanges. This is possible when market dynamics are nonlinear, since arbitrage-free opportunities only exist under linear pricing [41]. This is the important lesson we learn from high moment analysis.

5. Conclusion

In summary, high moments representation provides a new tool for diagnosing dynamical instability. High (3rd to 5th) moments would rapidly rise one-quarter before and during the crisis. These phenomena indicate a turbulent market, which can be understood by herd behavior induced by interacting agents. High moments are insignificant in a calm market. Changing high moments (in a range of 1 – 1000 times of normal variance) in financial history show that the real market does not follow a stable distribution. Therefore, the static picture of distribution tails may distort the non-stationary nature of market instability and crisis. The dramatic rising of high moments provides a better signal of market instability than traditional measures, such as the price level changes or fat tails in a static distribution. The stochastic dynamics of changing high moments and regime switch can be fully described by a nonlinear BD process whose transition probability can be estimated from empirical data. We would discuss this issue elsewhere [36].

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