### **Endogenous Fluctuations and Statistical Nature in Macro Dynamics:**

## From Equilibrium Noise to Persistent Cycles<sup>\*</sup>

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#### I. Introduction: Bridge the Gap between Economics and Biology

Alfred Marshall once remarked that economics should be considered closer to biology than mechanics (Marshall 1920). Living systems have two essential features: life rhythms, and the birth-death process. However, the current economic framework is far from Marshall's dream: Economic order is widely formulated by a steady-state solution plus random noise. Can we bridge the gap between equilibrium economics and evolutionary biology?

There is a fundamental problem in equilibrium economics: the nature of persistent business cycles. To study the problem, there are two different perspectives in economic dynamics: the equilibrium-mechanical approach and the evolution-biological approach.

The existence of persistent business cycles and chronic excess capacity is hard to explain by using equilibrium models in macro econometrics: External noise cannot maintain persistent cycles in the Frisch model (Chen 1999); Aggregate fluctuations in the Lucas microfoundations model are too weak for generating large macro fluctuations according to the Principle of Large Numbers (Lucas 1972, Chen 2002); Random walk and

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Brownian motion are not capable of explaining persistent fluctuations in macro indicators (Chen 2001).

In our analysis, the time scale plays a key role in understanding economic dynamics. The birth-death process is the first approximation of growth fluctuations. Business cycles can be further decomposed into a smooth trend, plus color chaos and white noise. Persistent cycles and structural changes can be directly observed from a time-frequency representation. Resilient market and economic complexity can be understood from persistent business cycles and technological metabolism. Economic evolution and structural changes can be directly observed from a wide range of time scales, including product cycles, business cycles, and Kondratieff long waves.

# II. Endogenous Fluctuations and Statistical Nature in Macro Dynamics:

#### From Equilibrium Noise to Persistent Cycles

The nature of business cycles is an unsolved issue in macroeconomics. There are two schools of thought in business cycle theory: the exogenous-shocks-equilibrium school and the endogenous-cycles-disequilibrium school.

The exogenous school has four pillar models: the Frisch model of a noise-driven damped oscillator, the Lucas microfoundations model of rational expectations, the random walk and the Brownian motion model in macro and finance theory (Frisch 1933, Lucas 1981, Nelson and Plossor 1982, Black and Scholes 1973). Endogenous cycles are represented by deterministic oscillators including harmonic cycle, limit cycle, and color chaos (Samuelson 1939, Goodwin 1951, Chen 1988).

In this section, we will show that equilibrium models are not capable of explaining large fluctuations and persistent cycles in macro movements. The thought experiments that argue against the existence of economic chaos have fundamental flaws in theoretical thinking. The birth-death process and color chaos model provide a better picture of market resilience and the economic clock observed from business cycles.

## (2.1) The Copernicus Problem in Macro Econometrics: Linear and Nonlinear Trends in Macroeconomic Indexes

The non-stationary feature of economic growth imposes a great challenge to theoretical economics and economic physics: How to identify some stable patterns from an evolving economy? Can we simplify the observed complex movements into some simple patterns by means of mathematical mapping? This is the Copernicus problem in macro econometrics. The time scale plays a critical role in observing business cycles.

Measurement and theory cannot be separated from each other. The dynamic patterns from competing observation references can be seen in Fig. 1.

In econometrics, the linear filter of first differencing (FD) is widely applied to construct an equilibrium (short-term) picture of economic fluctuations. The resulting time series are erratic and short-correlated (Fig. 1b). The random walk model with a constant drift is also called the unit-root model in macro econometrics (Nelson and Plosser 1982).

In neo-classical growth theory, the long-term equilibrium path is characterized by an exponential growth or a log-linear (LL) trend (Solow 1956). The resulting cycles are long-correlated. The problem is that measurement is sensitive to the choice of time boundaries.

An intermediate trend between FD and LL is a nonlinear smooth-trend obtained by the HP (Hodrick-Prescott) filter in the real business cycle literature (Hodrick and Prescott 1981)<sup>\*\*</sup>. Its correlation time is in the range of NBER business cycles of several years (Chen 1996a). We will show that the HP trend is better than the other two in giving a consistent picture of medium-term business cycles. This finding reveals the critical role of the time scale in choosing a preferred reference system.



(a). The HP trend and LL trend of X(t), the logarithmic S&P time series



(b). Autocorrelations of detrended cycles.

<sup>\*\*</sup> Prescott told the author at the AEA2001 meeting that the HP filter was first used by John von Neumann. So, a more accurate name is the VHP filter.

Fig. 1. Three detrending references and their auto-correlations of detrended cycles from the logarithmic SPX, the Standard and Poor 500 Price Index monthly series (1947-2002), N=672. Data source: yahoo.finance.

From Fig. 1b, we can see that the short correlations of FD series look random but HP cycles have an image of damped cycles, which could be generated by color noise or color chaos. Color means a characteristic frequency from the observed fluctuations. The observed variances also depend on the choice of the observation reference trend: the longer the time window, the larger the variance. The LL indicates the largest time-window of the entire observational period. The FD implies the shortest time-window of one time unit when macroeconomic trends are completely ignored. FD is the root of equilibrium illusion in macro econometrics. The HP implies a medium time-window in the range of business cycles.

## (2.2) Equilibrium Illusion in Business Cycle Theory: The Challenge of Large and Persistent Fluctuations

The four pillar models in equilibrium theory of business cycles have analytical solutions, which have fundamental difficulties in understanding persistent business cycles. The popular belief in an efficient market would be in trouble when economic complexity exists.

#### (2.2.1) Frisch Fantasy of Noise Driven Cycles:

A Perpetual Motion Machine of the Second Kind?

Frisch realized that the linear model has marginal stability in parameter changes [We will further discuss this issue in section (3.3.1)]. He speculated that persistent cycles could be maintained by a stream of random shocks. He claimed this scenario in an informal conference paper (Frisch 1933). Equilibrium economists quickly embraced the Frisch model because the stable nature of a market economy could be preserved by a damped harmonic oscillator with friction. However, the Frisch speculation was rejected by physicists in their study of the harmonic Brownian motion, which was solved analytically before Frisch (Uhlenbeck and Ornstein 1930).

The conclusion in physics is contrary to the Frisch fantasy: the harmonic oscillation under Brownian motion will be dampened in an exponential way. Persistent cycles cannot be maintained by random shocks. The relaxation time  $T_b$  and realized

period  $T_r$  can be estimated from observed autocorrelations:

$$\boldsymbol{r}(\boldsymbol{t}) = \exp(-\frac{\boldsymbol{t}}{T_b})\left[\cos(\frac{2\boldsymbol{p}\,\boldsymbol{t}}{T_r}) + \frac{T_r}{2\boldsymbol{p}T_b}\sin(\frac{2\boldsymbol{p}\,\boldsymbol{t}}{T_r})\right]$$
(2.1)

For the Brownian oscillator model of the logarithmic US real GDP, the estimation of relaxation time depends on the choice of the observation reference system. American business cycles would cease within 4 years for FD or 10 years for HP cycles respectively (Che n 1999). The FD reference is worse than the HP reference, since the FD cycles need a large source of external noise, whose standard deviation should be 30 % larger than the standard deviation of the US real GDP. Since the US economy is the largest in the world, we could not identify an external source to drive American business cycles. Clearly, a linear oscillator is not capable of modeling persistent cycles.

Historically, Frisch quietly abandoned his model as early as 1934. Frisch's promised paper, "Changing harmonics studied from the point of view of linear operators and erratic shocks," was advertised three times under the category "papers to appear in early issues" in *Econometrica*, including Issue No. 2, 3, and 4 of Volume I (April, July, and October 1933). The promised paper was never published in *Econometrica* where Frisch himself was the editor of the newly established flagship journal for the Econometric Society. Surprisingly, Frisch never mentioned a word about his prize-winning model in his Nobel speech in 1969 (Frisch 1981).

If Frisch could use random shocks to generate persistent cycles, it implies a perpetual motion machine of the second kind, which violates the second law of thermodynamics.

#### (2.2.2) The Lucas Issue of Microfoundations and the Principle of Large Numbers

The new classical school called for microfoundations of macroeconomic fluctuations. Lucas suggested that independent fluctuations at the level of households or firms (e.g., the inter-temporal substitution between work and leisure) would generate large fluctuations at the aggregate level (Lucas 1972, 1981). He simply ignored the essential differences between the one-body/many-body problem.

As a first approximation, we may consider a macro economy as a static system with N identical agents. The macro economy can be described by their total output. We assume that fluctuations in a firm's output or a household's working hours follow an identical independent distribution. The mean is  $\boldsymbol{m}$ , the standard deviation is  $\boldsymbol{s}$ . Based on the law of large numbers and the central limit theorem in probability theory, the mean of the aggregate positive output is  $N\boldsymbol{m}$ , while its variance is  $N\boldsymbol{s}^2$ . Therefore, we can define the relative deviation (RD= $\Psi$ ) by the ratio of the standard deviation to the mean when the mean of positive variables is not zero:

$$\Psi = \frac{\sqrt{VAR[S_N]}}{mean[S_N]} = \frac{C}{\sqrt{N}} \quad \text{where } C = \frac{s}{m}$$
(2.2)

For a nonstationary process with internal fluctuations, a linear birth-death process for economic growth will generate similar results (Chen 2002). We can define an implied number N\*, which can be estimated from the observed macro series:

$$N^* = \frac{1}{\Psi_{macro}^2} = \frac{\boldsymbol{m}_{macro}^2}{\boldsymbol{s}_{macro}^2}$$
(2.3)

We can say that the relative deviation for aggregate fluctuations of N statistically

independent positive elements is in the order of  $\frac{1}{\sqrt{N}}$ : we call this rule based on the law

of large numbers and the central limit theorem *The Principle of Large Numbers*. The relative deviation is a very useful measure for a wide class of systems with a positive range of variables, such as population, output, working hours, and price.

Empirical measurement of the relative deviation depends on the reference system in observing business cycles. Here, the relative deviation is measured by the ratio of the standard deviation of the HP cycles to the mean of the HP trends within a moving time window since the HP reference produces the largest implied numbers, which are compatible with empirical facts (Table I). Other references have even worse results.

Table I. The Relative Deviation and Implicit Number of Degrees of Freedom For Several Macro Indexes by HP detrending (1947-2000)

Tor Several Maero Indexes by The detending (1947-2000)				
$\Psi$ (%) [N*]	GDPC1Ln	PCECC96Ln	GPDIC1Ln	LBMNULn
HP	0.21[200,000]	0.17[300,000]	1.3[6,000]	0.29[100,000]

Here GDPC1 is the US real gross domestic product in 1996 US dollars; PCECC96, the real personal consumption; GPDIC1 the real domestic investment; and LBMNU the hours of non-farm business. The estimates of relative deviations are averages over the period from 1947-2000 with logarithmic data series.

The magnitude of the relative deviation of macro indexes is in the range of 0.2 to 1 percent; its implied number is between 200,000 and 6,000. How can we associate these figures with the actual numbers in the US economy? According to the U.S. Bureau of Census, there were 81 million households, 3 million corporations with more than \$100,000 in assets, and about 20 thousand public companies in 1980. If we compare these numbers with the implied numbers under HP trends, we can see that the observed implied numbers of these macro indexes are several hundred smaller than household or firm numbers. In another words, the observed relative fluctuations are at least 20 times larger than could be explained by the microfoundations models in labor or producer markets.

There are several implications from comparisons of these numbers.

First, the representative model in the real business cycle theory is not valid, since the

observed implied numbers are much larger than one.

Second, fluctuations in households or firms are not capable of explaining large relative deviations in aggregate output, consumption, business hours, or investment.

Third, financial intermediaries and industrial organizations appear to play a critical role in generating large business fluctuations since the number of large companies and large financial corporations matches the quantitative range required by implied numbers in investment.

A further examination of the Lucas model of inter-temporal substitution between goods and leisure reveals fundamental flaws in equilibrium thinking. In the Lucas island economy, identical agents believe and act in perfect correlation under rational expectations. If these agents have individual freedom of choice, arbitrage activity will eliminate correlations among individual fluctuations. Lucas claimed that government policy was effective only when it was unexpected. Similarly, rational expectations cannot last long if they mislead believers! Diversified choices are driven by conflicting interests rather than a common belief in a competitive but unequal society. *The rational expectations hypothesis suffers the same self-defeating syndrome of macro econometrics under the Lucas critique*. Clearly, the efficient market, rational expectations, and microfoundations theories do not provide a consistent framework for business cycle theory. They are, in fact, contradictory in explaining large business fluctuations.

#### (2.2.3) The Endogenous Mechanism and Statistical Property:

The Birth-Death Process against the Brownian Motion and Random Walk

In modeling stochastic growth, the exogenous school is based on the drifted diffusion model, which is also called the geometric Brownian motion model in finance theory [Black and Scholes 1973]. Two stochastic models of endogenous fluctuations are used in the economic theory of growth and fluctuations: the random walk model and the birth-death process [Nelson and Plossor 1982, Chen 2002]. It is widely perceived that the three stochastic models have similar behavior. There is little doubt about the validity of geometric Brownian motion in economic dynamics.

In the previous section, the relative deviation (RD) plays a fundamental role in studying microfoundations. RD is quite stable for observed macro indicators. Here, we further compare the relative deviations for three popular stochastic models of growth and fluctuations. Their analytical results are shown in Table II (Chen 2001, Li 2002).

Order	Drifted Diffusion	Birth-Death	Random-Walk
Mean	$\sim \exp(rt)$	$\sim \exp(rt)$	~t
Variance	$\sim \exp(2rt)\{e^{s^2t}-1\}$	$\sim e^{rt}(e^{rt}-1)$	~t

Table II. The Statistical Properties of Linear Stochastic Processes

RD	$\sim e^{\frac{s^2}{2}t} \sqrt{(1-e^{-ts^2})}$	$\sim \frac{1}{\sqrt{N_0}}$	$\sim \frac{1}{\sqrt{t}}$
	V V	$\sqrt{N_0}$	$\sqrt{t}$

Here,  $N_0$  is the size of initial population of micro agents in the birth-death process and r>0 for economic growth.

From Table II, we can clearly see that both the random walk and the Brownian motion model cannot generate sustained fluctuations: *the random walk is damping* and *the diffusion model is exploding in time*. It is interesting to note that these two models are a representative agent model in nature. The persistent pattern in economic fluctuations could only be explained by the birth-death process, which is a population model of growth and an endogenous model of fluctuations. This result raises a fundamental challenge to equilibrium models in terms of representative agent and exogenous fluctuations in macroeconomics and finance theory.

#### (2.2.4) Monetary Neutrality and Coordination Cost:

Ricardo Device, Loschmidt Paradox, and Uneven Distribution

The Ricardo device is a thought experiment to justify the neutrality of money. Here, thought experiments are named by their authors. The Ricardo device is the hypothetical operation of doubling overnight the cash holdings of all business enterprises and households without changing relative prices. It means that all supply and demand functions are a homogeneous function of zero degree, which is the basic argument against Keynesian economics (Leontief 1936). Ricardo ignored the redistribution problem in an unequal society. The Ricardo operation implies a legislation of progressive subsidy or regressive taxation, which has no chance of winning in parliamentary politics. The Ricardo device can only work in a primitive economy with an even distribution of wealth.

The Ricardo device in economics is very similar to the Loschmidt reversibility paradox for challenging Boltzmann's H theorem of thermodynamic irreversibility (Brush 1983). Loschmidt argued that one should be able to return to any initial state by merely reversing all molecules velocity under Newton's law. The trouble here is the huge coordination costs. As noted by Boltzmann in 1877, the possibility of reversing all the initial conditions is very unlikely in dealing with a large system with many particles. The empirical and theoretical evidence of monetary chaos is a challenge to the neutrality of money (Barnett and Chen 1988, Chen 1988). Our finding may revitalize the Austrian theory of endogenous money.

#### (2.2.5) Rational Arbitrager and Non-Replicate Patterns:

Friedman Spirits, Maxwell Demon, and Information Ambiguity

Friedman spirits are rational arbitrageurs who wipe out any destabilizing traders on a speculative market (Friedman 1953). The implication is that no structures can exist in a

competitive market, which is the main argument for the efficient market hypothesis.

Friedman spirits behave much like the Maxwell demon in equilibrium thermodynamics. The Maxwell demon is an imaginary gatekeeper trying to create a non-equilibrium order from an equilibrium state by operating a frictionless sliding door between two chambers that are filled with moving molecules. Maxwell assumed that his demon had perfect information about the speed and position of all molecules such that he could only allow a fast molecule into a designated portion by opening or closing the mass-less valve in perfect timing. Therefore, by utilizing information in a smart way, the Maxwell demon could create a temperature difference without doing work, that outcome is contrary to the second law of thermodynamics. No information cost is essential for its operation.

Friedman spirits face a similar problem as the Maxwell demon but in an opposite situation. To eliminate any market instability, Friedman spirits need perfect information and unlimited resources. However, informationally efficient markets are impossible because of the information cost (Grossman and Stiglitz 1980). Under financial constraints, the Friedman spirits may give up negative feedback strategy by following the mass psychology to avoid arbitrage risk, which results in creating instability (De Long et al 1990).

There is an even greater problem of information ambiguity. Friedman assumed that a winner's imitator could quickly replicate the winning pattern and drive down the profit margin to zero. This scenario could be true only if the destabilizing pattern were replicable. This is unlikely because of imperfect information (having only finite data with significant noise and time delays), information ambivalence (in face of conflicting news and mis-information), unpredictable events (such as a financial crisis and changing structure), and limited predictability (existence of deterministic chaos or wavelet). The critical issue of information ambiguity is not only associated with bounded rationality but also rooted in dynamical complexity (Simon 1957, Chen 1993a).

## (2.3) Living Rhythms and Economic Organisms: Color Chaos versus White Noise

The controversy of noise vs. chaos reveals the limitations of numerical tests in parametric econometrics and nonlinear dynamics (Chen 1988, 1993a, Brock and Sayers 1988, Benhabib 1993). Testing deterministic chaos in a non-stationary economic time series is more difficult than that of stationary data in laboratory experiments. Conventional econometrics could detect nonlinearity but not chaos. The critical issue here is finding a proper representation in dealing with a non-stationary economic time series.

In this section, we will introduce the Wigner transform in Gabor space for separating noise and cycles. We found abundant evidence of color chaos, which is similar to a biological clock.

(2.3.1) The Uncertainty Principle and Gabor Wavelet

The uncertainty principle in time and frequency is the very foundation of signal processing:

$$\Delta f \ \Delta t \ge \frac{1}{4p} \tag{2.4}$$

Here, f is the frequency and t is time. Minimum uncertainty occurs for a harmonic wave modulated by a Gaussian envelope, which is called the Gabor wavelet in signal processing or a coherent state in quantum mechanics. This is the very foundation of time-frequency analysis in the two-dimensional time-frequency Gabor space.

#### (2.3.2) Separating Noise and Cycles in Time-Frequency Space

For analyzing a time-dependent series, we introduced a new analytic tool, the joint time-frequency analysis (Qian and Chen 1996, Chen 1996a,b). A time-varying filter in a two-dimensional time-frequency lattice space can be applied for separating cycles and noise. Its localized bases are the Gabor wavelets. The filtered and unfiltered HP cycles are shown in Fig.2. The deterministic pattern of filtered HP cycles can be clearly seen from the phase portrait in Fig.3.



Fig. 2. The filtered FSPCOM (S&P 500 Price Index) HP cyclic series Xg closely resembles the original time series Xo. The correlation coefficient between Xg and Xo is 0.85. The ratio of their variance is 69 %. The correlation dimension of Xg is 2.5. Data source: Citibase.



Fig. 3. The phase portraits of the unfiltered (the left plot) and the filtered (the right plot) FSPCOM HP cycles. The time delay T is 60 months.

The phase portrait of filtered FSPCOM HP cycles shows a clear pattern of deterministic spirals, a typical feature of color chaos. Color chaos here refers to the nonlinear oscillator in continuous time. Color shows a strong peak in Fourier spectrum in addition to a noisy background (Chen 1996b).

#### (2.3.3) Natural Experiments of Economic Clock:

Intrinsic Instabilities and External Shocks in Evolving Economies

According to new classical economists, business cycles are all alike if they are generated by pure stochastic processes (Lucas 1981). From new observations in time-frequency analysis, we find that business cycles are not all alike because of strong deterministic components. The time-frequency patterns of macroeconomic indicators resemble biological organisms with multiple rhythms. The frequency path can reveal valuable information in economic diagnostics and policy studies (Chen 1996b).

Our picture of an economic clock is a dramatic contrast with those of a random walk in equilibrium economics. Can we conduct some out-of-sample tests to distinguish these two approaches? Perhaps not, because nonstationarity is the main obstacle to the application of statistics. However, the natural experiments of the oil price shock and the stock market crash demonstrate that time-frequency representation reveals more information than white-noise representation (Fig. 4).



Fig. 4. The time path of the basic period Pb of FSPCOMIn (the S&P 500 Price Index) HP cycles stock market indicators. The basic period  $P_b$  shifted after the oil price shock in October 1973, which signaled an external shock. In contrast, the frequency changes occurred before and after the stock market crash in October 1987, which indicated an internal instability during the crash.

Our finding of persistent cycles supports the biological view of business cycles (Schumpeter 1939).

In addition to stock market indexes, persistent cycles are widely observed from HP detrended economic aggregate indicators, including the gross domestic product, consumption, domestic investment, long-term interest rates, monetary supply indexes, the velocity of money, the consumer price index, and the unemployment rate (Chen 1996a). The range of their characteristic period is from two to ten years, a common feature of NBER business cycles. The noise component ranges from 20 to 50%. Certainly, not all macroeconomic indicators behave like biological clocks. Short-term interest rates and foreign exchange rates are very noisy. This information provides a valuable guide for macroeconomic study.

The frequency stability of economic indicators is remarkable. Surprisingly, market resilience is quite robust since most characteristic frequencies are very stable under external shocks and internal instabilities. The stock market crash in October 1987 led to a 23.1 percent drop in the level of the S&P 500 index in two months, but only a 6-percentage shift in its characteristic period.

The existence of persistent cycles within business fluctuations is strong evidence of economic color chaos. We should point out that the term of "chaos" has a negative image of disorder. We use the term of "color chaos" which adds a *life rhythm* to a nonlinear oscillator in continuous-time. Color means a characteristic frequency, which is similar to a biological clock. This is a contrast to white noise, which has no characteristic frequency.

(2.3.4) Structural Instability and Market Resilience

The structural stability of a market economy is hard to explain within the framework of linear dynamics. This problem can be demonstrated by the Samuelson multiplier-accelerator model (Samuelson 1939). The structural instability of the periodic mode in the Samuelson model can be seen in parameter space (Fig. 5).



Fig. 5. Stability pattern of Samuelson model in parameter space. Here, ST denotes the steady state; DO, damped oscillation; EO, explosive oscillation; EP, explosive solution; PO, linear periodic oscillation.

We can see that *the periodic regime PO has only a marginal stability* on the borderline between DO and EO. A small deviation from PO in parameter space will lead to damped or explosive oscillations. *The unit-root model in econometrics has similar marginal stability at the unit circle* (Nelson and Plosser 1982). The problem of structural instability is common for linear models. In the real world, a market economy is very resilient under various shocks.

The problem of structural instability in linear models could be solved by nonlinear models. Consider the example of the soft-bouncing oscillator or "freeway model," which is a mixed difference-differential equation with a targeted floor and ceiling (Chen 1988). Overshooting is caused by time-delay  $\tau$  in feedback control:

$$\frac{dX(t)}{dt} = aX(t) - bX(t - t) e^{-[\frac{X(t - t)^2}{s^2}]}$$
(2.5)

Where X is the deviation from the target; t the time delay;  $\pm s$ , the targeted floor and ceiling. The soft nature of control targets is characterized by a non-polynomial control function.

We may consider the left-side of Eq.(2.5) as the rate of change in excess-supply, while the right-side has a linear supply function but a nonlinear demand function. Soft-boundaries can be observed in many economic mechanisms, such as monetary control and the target zone of an exchange rate. A color chaos model of the soft-bouncing oscillator has a unified explanation of structural stability and pattern changes (regime switch) (Fig. 6). Pattern stability can be maintained under external shocks as long as a parameter shift does not cross a regime boundary, since periodic and chaotic regimes have finite measures. A regime switch occurs when an attractor moves into another regime in parameter space. During a regime switch, a small deviation in a parameter may induce a dramatic jump in dynamical patterns. In other words, a quantitative change leads to a qualitative change in such a situation.



(a). Parameter space for Eq. (2.5).

(b) The expanded regime in (a).

Fig.6 Stability pattern in parameter space. Here, ST denotes the steady state; C1, C2, C3 are limit cycles of period one, period two, and period three respectively; CH, the chaos mode in continuous time. The complex regime CP is enlarged in (b) that includes alternative zones of limit cycles and chaos.

#### IV. Conclusion: Understanding Market Resilience and Economic Complexity

Equilibrium economics has not developed a consistent framework of economic dynamics. Microeconomics is a static theory, where there is no room for product cycle, market-share competition, strategic innovation, and over investment. Macroeconomic modeling is dominated by a linear stationary model of the representative agent, which cannot explain the persistent nature of business cycles. The critical issues of non-integrable systems and continuous-time are ignored by the regression approach in econometrics.

Based on nonlinear dynamic models of business cycles and division of labor, we are developing a biological framework of evolutionary economic dynamics. Staring from empirical analysis of the macroeconomic time series, we have abundant evidence about the endogenous nature of macroeconomic fluctuations. Based on our theoretical study, the birth-death process is a better alternative than the Brownian motion in the first approximation of stochastic growth (Ji 2003). Persistent cycles around the HP trend can be

further refined by the deterministic model of color chaos in the soft-bouncing oscillator. Product cycles and excess capacity resulted from market-share competition.

From the biological perspective, business cycles are living rhythms with structural stability and dynamic resilience. Product cycles and business cycles are in the same range of time scale, ranging from several months or several years to decades. Examples are short cycles in computer software and long cycles in railway construction. Disruptive technology changes are the driving force of persistent business cycles. This perspective will fundamentally change our views on economic systems and government policies. Compared to the Fourier (plane wave) transform and pulse (noise) representation, the Gabor wavelet representation (or coherent state in quantum mechanics) provides a better mathematical representation for evolutionary ecology and economic dynamics. We will discuss the development of economic physics elsewhere.

We summarize the difference between the equilibrium-mechanical approach and the evolution -biological approach in Table III.

Subject	Mechanical	Biological
Micro	Factor cost, marginal pricing	Prod. cycle, market-share competition
Macro	Stability & convergence	Un-even growth & persistent cycles
Math	Static optimization	Nonlinear dynamics
Microfound.	One-body (representative)	Many-body problem
Test	Regression, forecasting	Natural & lab experiments
History	Ergotic (no memory)	Path-dependence
Information	Perfect/imperfect info	Complexity, ambiguity
Decision	Individual rationality	Social learning
Risk	Risk-aversion	Risk-taking/risk-aversion
Strategy	Maximizing profit	Balancing opportunity/risk
Variable	Relative price	Market share
Order	Balancing demand-supply	Technology competition
Technology	Fixed parameter	S-curve
Innovation	Random shocks	Disruptive stages
Pricing	Price taker (perfect market)	Strategic pricing
	Price setter (monopolistic)	Profit margin & growth space
Div. of labor	Different commodities	Coexistence of technologies
Business cycles	External shocks	Endogenous cycles
Growth	Brownian motion	Birth-death and wavelets
Trend reference	FD, LL, HP	HP
Fluctuations	Linear cycles	Nonlinear color chaos
	+white noise	+ white/fractal noise

Table III. Concepts and Representations in Mechanical and Biological Approach

Prediction	Mechanical	Biological
Price	Convergence	Differentiation
Development	Convergence (Solow)	Rise and fall of organizations
	Learning by doing	Learning by trying
Macro Fluct.	Labor choice (Lucas)	Industrial competition
	Technology shocks (RBC)	Birth & death of technologies
	Money shocks (Friedman)	Excess capacity & over investment
	Labor rigidity (Keynes)	Structural & Financial Changes
Institution	Transaction costs	Trade-off between
	Coordination costs	stability and complexity
Socio-evolution	Convergence	Bifurcations in evolution tree
Policies	Laissez fair (classical)	Regulations & standard
Governments	Demand-side policies	Supply-side policies
Institution	Property rights	Strategies innovations

From the above discussion, we can see that these two approaches are complementary to each other to some degree. However, the mechanical approach can be considered a special situation within a more general framework of a biological approach, since a conservative system is a special case of the open system.

In macro issues, the evolutionary approach can be a synthesis of conflicting linear theories. For example, the new classical school and the Keynesian school can be integrated into a general macro dynamics including different time scales: the new classical school mainly on the long-run, Keynesians on medium-term, and financial economists on short-run. In our decomposition of macro indexes, the long-run is the nonlinear (HP) smooth trend, the medium-run is persistent cycles around the trend, and the short-run is residual noise imposed on persistent cycles.

In micro issues, the evolutionary approach may establish a link between cost analysis on the demand side and market-share analysis on the supply side. Empirical observations of product cycles and marketing strategies in business research may provide a solid foundation for micro dynamics.

From our understanding, "dissipative structure" and "self-organization" means a higher kind of order in living systems, such as living rhythms, metabolism, and complex systems. The classical concept of equilibrium and entropy only describe the order-less state without differentiated structure. If we compare self-organization in complexity science with the equilibrium order shaped by "friction" or "rigidity" in equilibrium economics, you may appreciate our new concepts of *market resilience* and *persistent cycles*. The central concept in cybernetics and economic science is "stability" under "negative feedback" (Wiener 1948). We propose the terms of *complexity* and *resilience* to combine the two

aspects within living systems and social systems: *stability* under small external shocks and *viability* under large environmental changes. Both features can be observed from parametric space in complex dynamics and stable regimes of time rhythms in WGQ representation.

The proposed trade-off theory between stability and complexity and the generalized Smith theorem may address a wide range of phenomena in living and social systems. We believe that economic science is facing a great transition from equilibrium thinking to an evolutionary paradigm. The efficacy of our approach to economic dynamics is open to future research and experiments.

Finally, I would like to quote James Buchanan regarding his prediction on the future of economics (Buchanan 1991):

"The shift toward emergent order as a central perspective will be paralleled by a corollary, even if not necessary, reduction of emphasis on equilibrium models. The properties of systems in dynamic disequilibrium will come to centre stage, and especially as economics incorporates influences of the post-Prigogine developments in the theory of self-organizing systems of spontaneous order, developments that can be integrated much more easily into the catallactic than into the maximizing perspective....."

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