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## **Demand Uncertainty, Vertical Structure and Risk**

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**Abstract:** Consider a market of a non-storable commodity with uncertain aggregate demand. Both upstream producers and downstream retailers are price-takers. The production sector has increasing marginal costs and the retail sector has constant marginal costs. Linear retail prices are determined before the demand uncertainty is resolved. It is shown that when firms are risk neutral, the vertical structure of the industry does not influence the equilibrium final prices. However, it does influence the profit variations. The upstream and downstream profits under vertical separation are negatively correlated with each other. Hence the separation exaggerates the risk faced by the firms.

**Keywords:** Demand Uncertainty; Electricity; Non-Storable Goods; Vertical Integration

**JEL Classification:** L22, L51, L94

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## I. Introduction

Under imperfect competition, vertical industrial structure may influence market outcomes because of the “externalities” of individual pricing strategies (Spengler, 1950; Rey and Stiglitz, 1988; and others). In particular, vertical structure often matters when demand is uncertain (Carlton, 1979; Deneckere, Marvel and Peck, 1996; Dana and Spier, 2001; Wang, 2004; and others). However, the literature does not offer a theory on whether vertical structure influences market outcome when both upstream and downstream markets are competitive, probably because the answer is too “straightforward”. This paper considers a model with increasing marginal production costs, constant average retail costs, and uncertain aggregate demand. The retail prices must be determined before the demand realizes. All firms are price-taking and risk neutral. Two points are presented. First, vertical structure does not influence firms’ expected (short-run) profits or final prices. Second, vertical separation leads to more variations in profits for firms.

It has been suggested in the literature that vertical separation increases the risk faced by firms. The separation leads to unsecured factor supply (Calton, 1979, and others) or strategic interaction between upstream and downstream firms (Williamson, 1985, and others). The argument is supported empirically by Helfat and Teece (1987). The present paper suggests another possible mechanism of the argument. If firms are price-takers and the industry is vertically separated, the upstream and downstream profits are negatively correlated. Hence the firms face more profit variations under vertical separation.

Industrial organization theorists have paid considerable attention to vertical separation under demand uncertainty. The models usually assume that outputs are produced in advance

and inventoried for possible sale. Unsold inventories are wasted or devalued. Hence the products in consideration are “non-storable” but can be stored for one period. Demand uncertainty often leads to insufficient retail inventories, because retailers concern about being saddled with unsold units. Vertical integration or restraints can often restore efficiency. In contrast, the present paper assumes that production occurs after the uncertain demand realizes. Hence inventory holding is not necessary. Compared to the models in demand uncertainty literature, the role of product storability is less critical in our model. The idea can be extended to the cases where inventory holding is costly.

This paper considers a model where upstream sector has increasing marginal costs, downstream sector has constant average costs, aggregate demand is uncertain, and retail prices must be determined before the uncertainty resolves. It finds that although vertical structure does not influence the expected profits of the firms, it does influence the profit variation. The expected profits are more fluctuating under vertical separation. The analysis can also be viewed as comparing the market outcomes of fixed pricing and variable pricing in a vertically integrated industry. Indeed, from the perspective of producers, the market under vertical separation is equivalent to that under vertical integration with variable pricing. In both cases the producers’ selling prices equal their marginal costs.<sup>3</sup> Hence this paper suggests that in a vertically integrated industry, price-taking firms earn the same expected profits under the two types of pricing, but face more risk under variable pricing.

We use the electricity market as an example to present the ideas. Electricity is usually traded through contracts signed in advance. The demand for electricity is uncertain and

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<sup>3</sup> See Joskow (1976) for a review of marginal-cost (or peak load) pricing in electric industry.

periodic. However we ignore the issues related to market power, rationing protocols, government regulation, and some others. There is a theoretical literature on the competition of electricity markets. Klemperer and Meyer (1989) and von der Fehr and Harbord (1993) analyze the equilibria of oligopolistic markets where firms face uncertain demand. The firms compete by offering continuous (Klemperer et al., 1989) or discrete (von der Fehr et al., 1993) supply functions. Allaz and Vila (1993) suggest that the existence of future markets increases the efficiency of markets in a Cournot setting. Based on this theory, Bushnell, Mansur and Saravia (2008) simulate electricity prices that define bounds on static oligopoly equilibria, and find that vertically integrated wholesalers, or those with long-term contracts, have substantially less incentive to raise wholesale prices. In contrast to the oligopolistic models in the literature, the present paper considers a competitive model where firms are price-takers.

The rest of the paper proceeds as follows. Section II offers a simple model of perfect competition, which includes an upstream sector and a downstream sector. Section III characterizes the equilibria of the market, and Section IV discusses how the vertical industrial structure influences the risk faced by the firms. Section V discusses some possible extensions of the model. Section VI concludes the paper.

## **II. An Electricity Market**

An electricity market is perfectly competitive at both production and retail stages. All firms are price-taking and risk-neutral. The production cost of a producer (or generator) is  $C(q)$ , which satisfies

$$C(q) > 0, \quad C'(q) \equiv MC(q) > 0, \quad \text{and} \quad C''(q) \equiv MC'(q) > 0, \quad \text{for any } q > 0.$$

Since we only consider short run equilibrium, assume  $C(0) = 0$  without loss of generality. Retailers (“load serving entities” or “retail suppliers” for instances) have zero fixed costs and constant marginal costs. The marginal retail costs are also normalized to zero without loss of generality. We do not consider the role of transmission network in this paper.

There is a continuum of consumers. Their demands for the commodity are perfectly inelastic, homogenous, and uncertain.<sup>4</sup> The individual demands are perfectly correlated. The aggregate demand follows cumulative distribution  $F(\cdot)$  on interval  $[\underline{x}, \bar{x}] \subseteq R^+$ . The expected aggregate demand is denoted as  $N \equiv \int_{\underline{x}}^{\bar{x}} q dF(q)$ . Without loss of generality, the number of consumers is assumed to be  $N$ . Hence the demand of each consumer follows distribution  $F(Nx)$  on interval  $[\underline{x}', \bar{x}'] \equiv [\frac{\underline{x}}{N}, \frac{\bar{x}}{N}]$ , and her expected demand is 1.

The industry could be vertically integrated or separated. In both cases, we assume that the retail prices must be determined before the demand uncertainty resolves. The game played under vertical integration is as follows. First, given the market price, the integrated price-taking producers choose the number of consumers to sign supply contracts. A contract specifies a linear price, but not the quantity of transaction. Second, the demand uncertainty resolves and the producers satisfy the demand of the consumers at the predetermined price. The game played under vertical separation is as follows. First, given the market retail price, the retailers choose the numbers of consumer to sign supply contracts. Again, a contract only specifies a linear retail price. Second, the demand uncertainty resolves. The retailers purchase the commodity from the producer in a competitive spot wholesale market to meet the demand

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<sup>4</sup> Electricity consumers that are on traditional meters usually do not react to real-time prices. Wolak (2003) suggests that “...the retail market policies that currently exist in almost all states, including California, makes the hourly demand for electricity virtually insensitive to the value of the hourly wholesale price, particularly in the real-time energy market” (page 14).

of the consumers.

### III. The Market Equilibria

We normalize the number of price-taking producers to 1 in order to simplify the explanation. This simplification leads to no loss of generality as long as the equilibrium outcome is symmetric in firms. We will show that the model can be extended to the case with multiple producers at the end of this section.

#### 3.1 Vertical integration

Under vertical integration, the producer contracts directly with consumers. Because consumer demand is uncertain, the producer cannot specify production quantities in the contracts. However, the producer can choose the number of consumers to serve. The equilibrium of the market can be characterized by a price  $p^l$  that clears the market. If the firm signs up  $n$  consumers, its expected profit is

$$\pi(n) = np^l - \int_{\frac{n}{N}\underline{x}}^{\frac{n}{N}\bar{x}} C(q) dF\left(\frac{N}{n}q\right) = np^l - \int_{\underline{x}}^{\bar{x}} C\left(\frac{n}{N}q\right) dF(q).$$

The risk-neutral producer seeks to maximize the expected profit. Suppose that there is an interior solution. The first order condition of the problem is

$$\pi'(n) = p^l - \frac{1}{N} \int_{\underline{x}}^{\bar{x}} q MC\left(\frac{n}{N}q\right) dF(q) = 0.$$

Hence the optimal number of contracted consumers,  $n$ , is implicitly given by equation

$$p^l = \frac{1}{N} \int_{\underline{x}}^{\bar{x}} q MC\left(\frac{n}{N}q\right) dF(q).$$

Note that the price-taking producer views price  $p^l$  as given. It would serve more consumers when  $p^l$  is higher. Because the consumer's demands are perfectly inelastic, the equilibrium price is the lowest price that induces the producer to serve all the consumers. The market

clearing condition,  $n = N$ , implies that the equilibrium price is

$$p^* = \frac{1}{N} \int_{\underline{x}}^{\bar{x}} qMC(q)dF(q).$$

The expected profit of the vertically integrated producer is thus

$$\pi^I = Np^* - \int_{\underline{x}}^{\bar{x}} C(q)dF(q) = \int_{\underline{x}}^{\bar{x}} qMC(q)dF(q) - \int_{\underline{x}}^{\bar{x}} C(q)dF(q).$$

### 3.2 Vertical separation

Under vertical separation there is a competitive wholesale market. The spot wholesale price equals the marginal production costs.<sup>5</sup> Suppose that there are  $J$  independent price-taking

retailers, which have zero marginal costs. Denote the number of consumers signed up by

retailer  $j$  as  $n_j$ , and  $n \equiv \sum_{j=1}^J n_j \leq N$ . Hence the aggregate demand faced by the retailers,

denoted by random variable  $x'$ , follows distribution  $F(\frac{N}{n}x)$  on interval  $[\frac{nx}{N}, \frac{n\bar{x}}{N}]$ .

Retailer  $j$  purchases  $\frac{n_j}{n}x'$  units of the commodity from the spot market at wholesale price  $MC(x')$ , and sells to consumers at market-determined retail price  $r$ . Note that the wholesale price is decided by the aggregate demand. An individual price-taking retailer is unable to influence the aggregate demand or the wholesale prices. Given the market retail price  $r$ , retailer  $j$  maximizes the following expected profit

$$\begin{aligned} \pi_j^R(n_j) &= n_j r - \int_{\frac{nx}{N}}^{\frac{n\bar{x}}{N}} (\frac{n_j}{n}q)MC(q)dF(\frac{N}{n}q) = n_j [r - \int_{\frac{nx}{N}}^{\frac{n\bar{x}}{N}} (\frac{1}{n}q)MC(q)dF(\frac{N}{n}q)] \\ &= n_j [r - \frac{1}{N} \int_{\underline{x}}^{\bar{x}} qMC(\frac{n}{N}q)dF(q)]. \end{aligned}$$

Since a retailer's marginal cost is a constant (zero), it would sign up more consumers if

$$r > \frac{1}{N} \int_{\underline{x}}^{\bar{x}} qMC(\frac{n}{N}q)dF(q)$$

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<sup>5</sup> The Federal Power Act (1935) of the US imposed a statutory mandate on Federal regulator (FPC and FERC later) to set "just and reasonable" wholesale electricity prices. However, even in the absence of market power, the spot wholesale price could be extremely high or low, depending on the demand. It is difficult to tell whether a spot price is "fair and reasonable".

and vice versa. Neither case is an equilibrium outcome. Hence given retail price  $r$ , the equilibrium number of contracted consumers,  $n$ , is implicitly given by equation

$$r = \frac{1}{N} \int_{\underline{x}}^{\bar{x}} qMC\left(\frac{n}{N}q\right)dF(q).$$

Because the marginal production cost is increasing, the right side of the above equation is increasing in  $n$ . Hence the number of consumers that a retailer is willing to serve is increasing with the market retail price  $r$ . If the total number of contracted consumers is less than  $N$ , which means that demand exceeds supply, the retail price would be driven up, and vice versa.

Hence in equilibrium we have  $n = N$  and the equilibrium retail price is

$$r^* = \frac{1}{N} \int_{\underline{x}}^{\bar{x}} qMC(q)dF(q).$$

which is the same as the final price under vertical integration. We write the result as follow.

**Proposition 1:** *In the competitive market with uncertain demand, the vertical structure does not influence the equilibrium final prices, i.e.,  $p^* = r^*$ .*

The retailers always make zero expected profits because of their constant average costs.

The upstream producer's expected profit is the difference between expected revenue and expected cost, i.e.,

$$\pi^S = \int_{\underline{x}}^{\bar{x}} qMC(q)dF(q) - \int_{\underline{x}}^{\bar{x}} C(q)dF(q).$$

It equals the producer's expected profit under vertical integration.

**Corollary 1:** *In the competitive market with uncertain demand, the vertical structure does not influence the equilibrium profit of the upstream producer, i.e.,  $\pi^I = \pi^S$ .*

### 3.3 Multiple competitive producers

One might concern about the assumption that there is only one price-taking producer. This simplification is actually innocent as long as the producers are homogenous. The analyses can be easily extended to the case with multiple homogenous producers. Suppose there are  $m$  competitive producers. Each has a cost function of  $C(q)$ . The games played in the market are virtually the same as before.

If a firm signs up  $n$  consumers, its expected profit is

$$\pi(n) = np^I - \int_{\frac{n}{N}\bar{x}}^{\frac{n}{N}\bar{x}} C(q)dF\left(\frac{N}{n}q\right) = np^I - \int_{\bar{x}}^{\bar{x}} C\left(\frac{n}{N}q\right)dF(q).$$

The first order condition for the profit-maximization problem is

$$\pi'(n) = p^I - \frac{1}{N} \int_{\bar{x}}^{\bar{x}} qMC\left(\frac{n}{N}q\right)dF(q).$$

The optimal number of contracted consumers is implicitly given by equation

$$p^I = \frac{1}{N} \int_{\bar{x}}^{\bar{x}} qMC\left(\frac{n}{N}q\right)dF(q).$$

Because the consumers' demands are perfectly inelastic, the equilibrium price is the lowest price that induces the producer to serve all the consumers. The market clearing condition,  $n = \frac{N}{m}$ , implies that the equilibrium price is

$$p^{I*} = \frac{1}{N} \int_{\bar{x}}^{\bar{x}} qMC\left(\frac{q}{m}\right)dF(q).$$

The expected profit is thus

$$\pi_i^{I*} = \frac{1}{m} \int_{\bar{x}}^{\bar{x}} qMC\left(\frac{q}{m}\right)dF(q) - \int_{\bar{x}}^{\bar{x}} C\left(\frac{q}{m}\right)dF(q).$$

Under vertical separation, the retailer's problem is unaffected when there are multiple producers. The equilibrium retail price is still

$$r = \frac{1}{N} \int_{\underline{x}}^{\bar{x}} q MC\left(\frac{n}{N}q\right) dF(q).$$

Note that the spot wholesale price is  $MC\left(\frac{q}{m}\right)$  and each producer's output is  $\frac{q}{m}$ . Hence the equilibrium profit of a producer is

$$\pi_i^{S*} = \int_{\underline{x}}^{\bar{x}} \frac{q}{m} MC\left(\frac{q}{m}\right) dF(q) - \int_{\underline{x}}^{\bar{x}} C\left(\frac{q}{m}\right) dF(q) = \pi_i^{I*}.$$

We see all the analyses are parallel.

#### IV. *Ex post* Profit and Risk

Under vertical integration, given a final price  $p^I \in (MC(\underline{x}), MC(\bar{x}))$ , there exists a output level  $q'$ , such that  $MC(q') = p^I$ . The producer's *ex post* profit, as a function of the aggregate demand  $x$ , is<sup>6</sup>

$$\tau^I(x) = p^I x - C(x) = \int_0^x (p^I - MC(q)) dq.$$

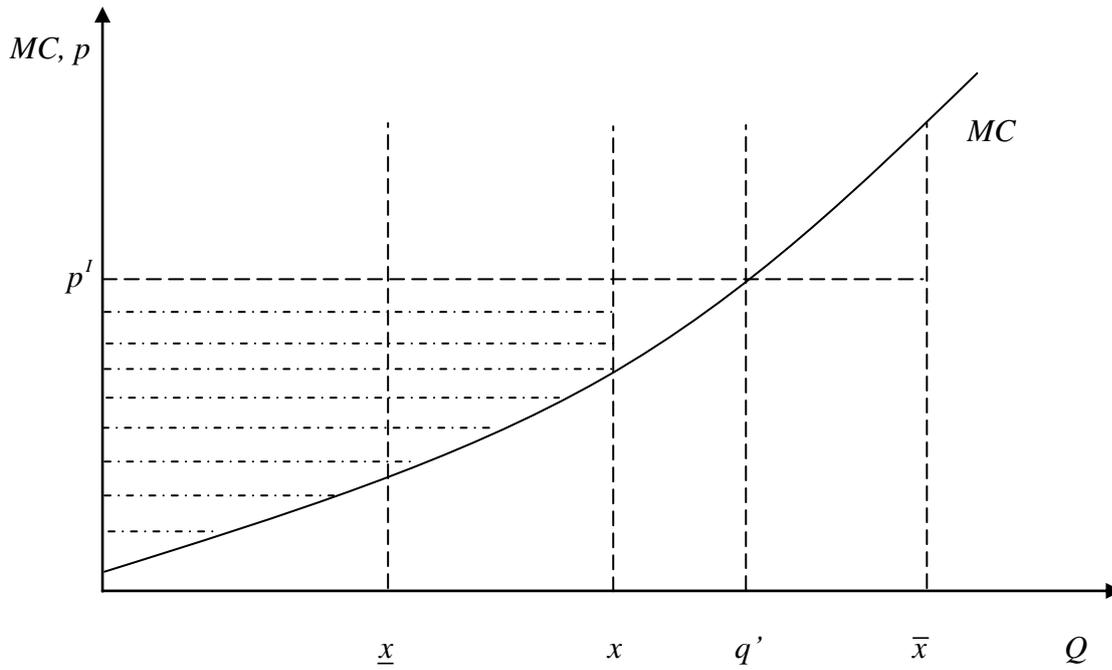
When  $x > q'$ , we have  $MC(x) > p^I$ , which means that the producer's last unit of output incurs a loss. Otherwise when  $x < q'$ , the last unit incurs a gain.

When  $x \leq q'$ , the *ex post* profit can be represented by the shadowed area of Figure 1;

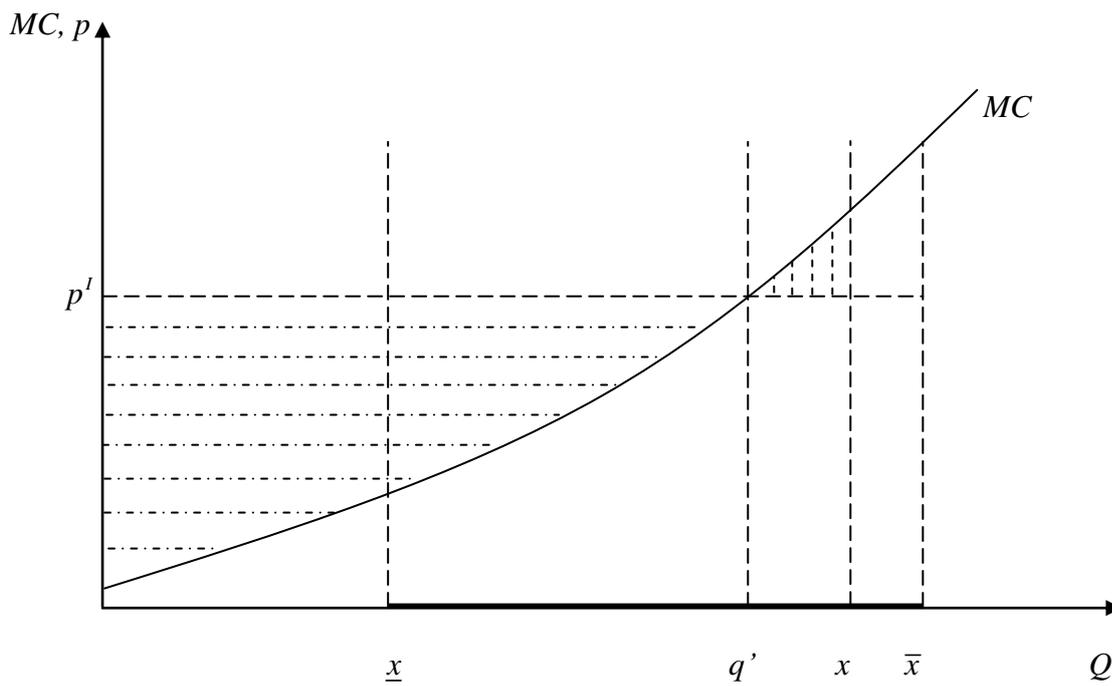
When  $x > q'$ , the profit can be represented by the left shadowed area net of the right shadowed area of Figure 2.

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<sup>6</sup> Note that the fixed cost  $C(0)$  is assumed to be zero.



**Figure 1:** *Ex post* upstream profit under vertical integration (when  $x \leq q'$ )



**Figure 2:** *Ex post* upstream profit under vertical integration (when  $x > q'$ )

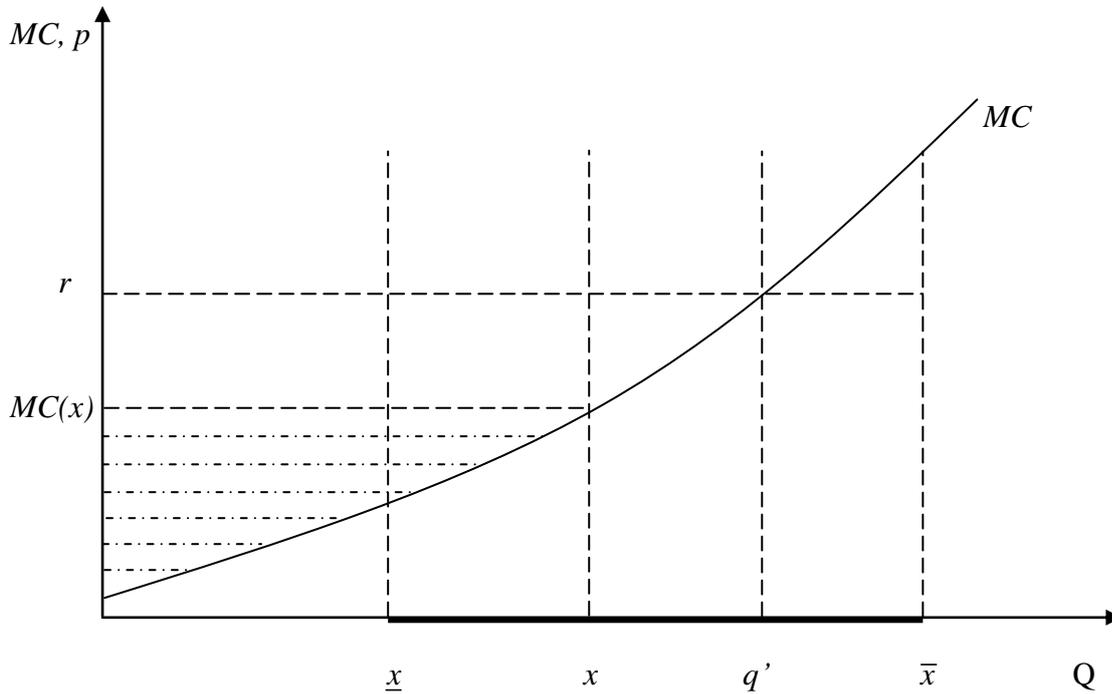
Under vertical separation, given a final price  $r$ , the producer and retailers' *ex post* profits, denoted by  $\tau_p^s(x)$  and  $\tau_r^s(x)$  respectively, are

$$\tau_p^S(x) = MC(x)x - C(x) = MC(x)x - \int_0^x MC(q) dq = \int_0^x (MC(x) - MC(q)) dq$$

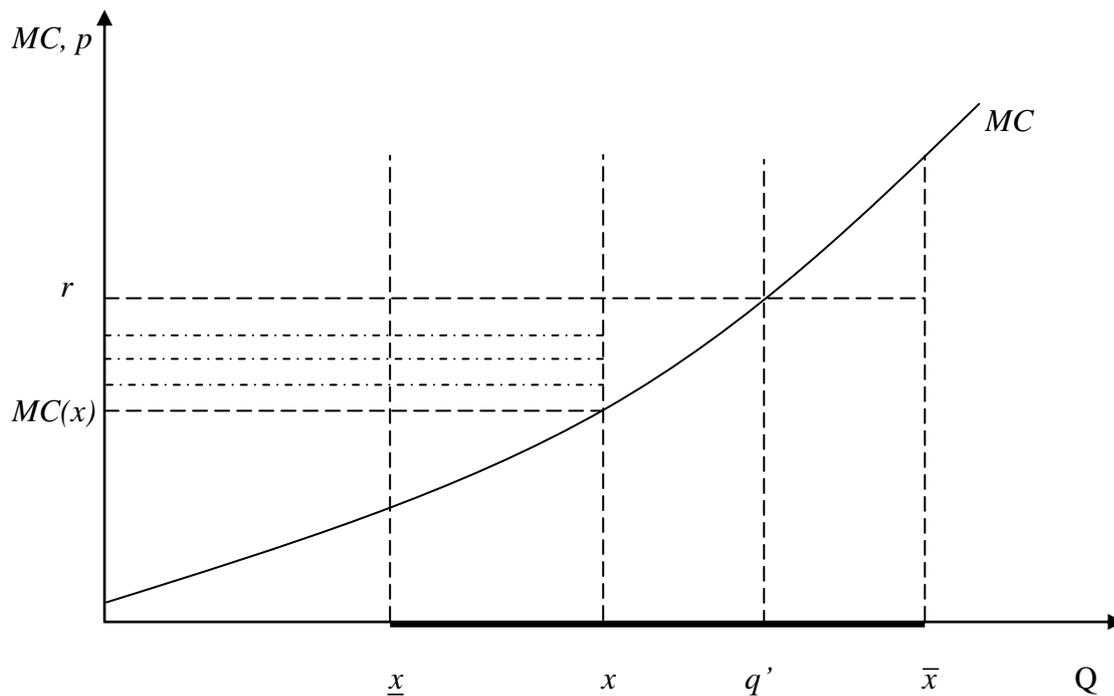
$$\tau_r^S(x) = [(r - MC(x))x] = \int_0^x (r - MC(x)) dq.$$

Note that in equilibrium we have  $r = p'$ . Hence  $MC(q') = p' = r$ . The producer's *ex post* profit can be represented by the area below the wholesale price  $MC(x)$  and above the marginal cost curve, as shown by the shadowed area of Figure 3.

The retailers' *ex post* profit can be represented by the shadowed rectangular in Figure 4. It is positive when and only when  $x < q'$ . The trade-off faced by a retailer is that a higher demand is always accompanied by a higher wholesale price. When  $\underline{x}$  is close to  $q'$  enough, the *ex post* downstream profit is maximized at  $\underline{x}$ , i.e.,  $\arg \max \tau_r^S(x) = \underline{x}$ .



**Figure 3:** *Ex post* upstream profit under vertical separation



**Figure 4:** *Ex post* downstream profit under vertical separation

We have following lemmas regarding the equilibrium outcomes.

**Lemma 1:** The vertical structure does not influence the *ex post* total profit of the industry, i.e.,

$$\tau^I(x) = \tau_p^S(x) + \tau_r^S(x), \quad \forall x \in [x, \bar{x}].$$

**Proof:** 
$$\begin{aligned} \tau_p^S(x) + \tau_r^S(x) &= \int_0^x (MC(x) - MC(q))dq + \int_0^x (r - MC(x))dq \\ &= \int_0^x (r - MC(q))dq = \int_0^x (p^I - MC(q))dq = \tau^I(x). \end{aligned}$$

**Lemma 2:** Under vertical integration, the producer's *ex post* profit  $\tau^I(x)$  is maximized at  $x = q'$ .

**Proof:** Since  $\frac{d\tau^I(x)}{dx} = p^I - MC(x)$  and function  $MC(\cdot)$  is strictly increasing,  $\tau^I(x)$  is maximized when  $p^I = MC(x)$ , i.e.,  $x = q'$ .

**Lemma 3:** *The retailers' ex post profits are non-negative if and only if the aggregated demand is not greater than  $q'$ , i.e.,  $\tau_r^S(x) \geq 0$  if and only if  $x \leq q'$ .*

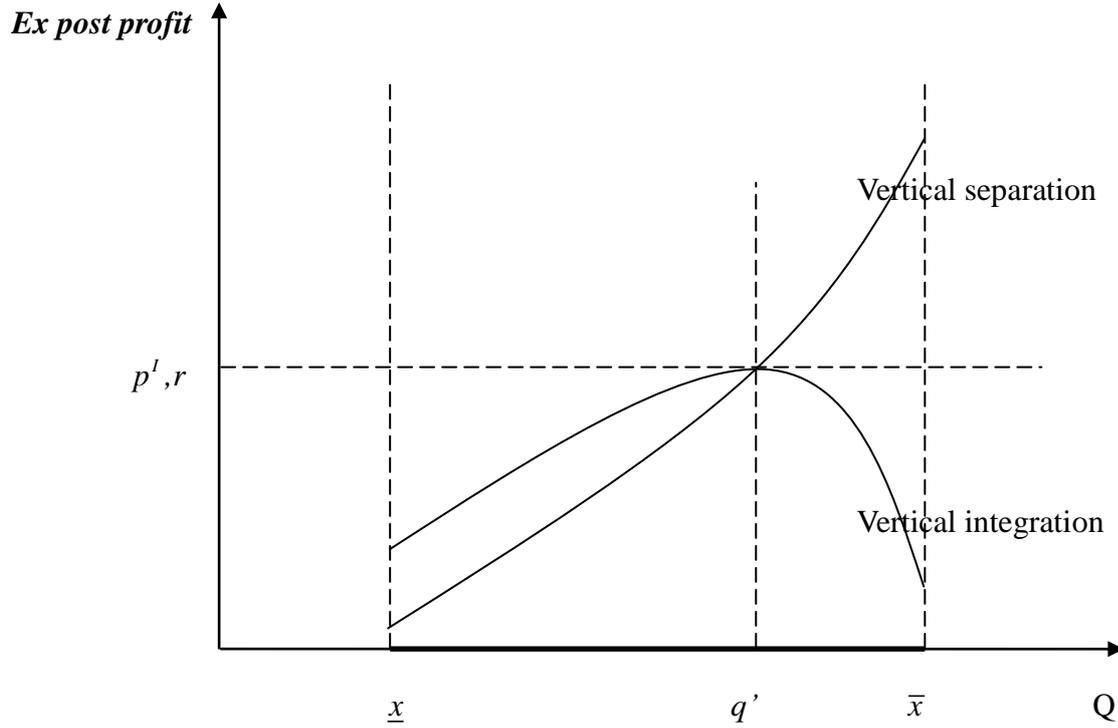
**Proof:** When  $x \leq q'$ ,  $MC(x) \leq r$ . Hence  $\tau_r^S(x) = [(r - MC(x))x] \geq 0$ , and vice versa.

From Lemma 1 we have  $\tau_r^S(x) = \tau^I(x) - \tau_p^S(x)$ . Hence Lemma 3 immediately implies that  $\tau^I(x) \geq \tau_p^S(x)$  if and only if  $x \leq q'$ .

**Lemma 4:** *Under vertical separation, the producer's ex post profit is increasing with the aggregate demand, i.e.,  $\frac{d\tau_p^S}{dx} > 0$  for any  $x \in [\underline{x}, \bar{x}]$ .*

**Proof:**  $\frac{d\tau_p^S}{dx} = MC'(x)x + MC(x) - MC(x) = MC'(x)x > 0$ .

Figure 5 depicts the upstream producers' *ex post* profits under the two market structures. The difference between the two profits is the downstream retailers' profits or losses. Most of the lemmas can be illustrated by this figure. We see from the figure that the producers' *ex post* profits fluctuate more under vertical separation. In particular, it is more likely for a producer to earn extraordinary high profits (when demand is high), which implies dangerous losses for retailers.



**Figure 5:** *Ex post* upstream profits

**Lemma 5:** *There exists  $\hat{x} < q'$ , such that  $\frac{d\tau_r^S(x)}{dx} < 0$  for any  $x \in (\hat{x}, \bar{x}]$ .*

**Proof:**  $\frac{d\tau_r^S}{dx} = [r - MC(x)] - MC'(x)x$ . Since  $MC(x)$  is increasing,  $MC(q') = r$  and  $MC'(x)x > 0$ , we have  $\frac{d\tau_r^S}{dx} < 0$  when  $x \geq q'$ . The lemma is obtained immediately by the continuity of the functions.

In an electricity market, the base load  $\underline{x}$  is often substantial and stable. Demand uncertainty takes the form of upward demand shocks, and the shocks are small in size compared to the base load. In that case, we shall have  $q'$  being close to  $\underline{x}$ , and thus  $\hat{x} = \underline{x}$  in Lemma 5.

**Lemma 6:** When  $\hat{x} = \underline{x}$  in Lemma 5,  $Cov(\tau_p^S(x), \tau_r^S(x)) < 0$ , i.e., the *ex post* profits of the upstream and downstream firms are negatively correlated.

**Proof:** Because  $\frac{d\tau_p^S}{dx} > 0$  and  $\frac{d\tau_r^S}{dx} < 0$ ,  $Cov(\tau_p^S(x), \tau_r^S(x)) < 0$  by Schmidt (2003).

**Proposition 2:** When  $\hat{x} = \underline{x}$  in Lemma 5, which means  $\frac{d\tau_r^S(x)}{dx} < 0$  for all  $x \in [\underline{x}, \bar{x}]$ , the sum of the variance of  $\tau_p^S(x)$  and  $\tau_r^S(x)$  is larger than the variance of  $\tau^l(x)$ .

**Proof:** We have

$$\begin{aligned} Var(\tau^l(x)) &= Var(\tau_p^S(x) + \tau_r^S(x)) \\ &= Var(\tau_p^S(x)) + Var(\tau_r^S(x)) + Cov(\tau_p^S(x), \tau_r^S(x)) \\ &< Var(\tau_p^S(x)) + Var(\tau_r^S(x)) \end{aligned}$$

The last step is by Lemma 6.

If we measure the “risk” faced by a firm by the variance of its *ex post* profit, then Proposition 2 suggests that the “aggregate risk” faced by the upstream sector and downstream sector are larger under vertical separation, because the upstream and downstream profits are negatively correlated. In particular, under vertical separation, when demand is high, the producer makes substantial profits while the retailers are losing money. This feature leads to exaggerated risks at both sides.

It is also possible for the upstream producer to take more risk under vertical separation. Suppose that the production cost is quadratic and thus the marginal cost is linear. Specifically, let  $C(x) = \frac{k}{2}x^2$ ,  $k > 0$ , and thus  $MC(x) = kx$ . The equilibrium price and *ex post* profits are

$$p^I = r = \frac{E(MC(x)x)}{E(x)} = \frac{E(kx^2)}{E(x)},$$

$$\tau_g^S(x) = MC(x)x - C(x) = \frac{k}{2}x^2,$$

$$\tau_r^S(x) = p^I x - C(x) = p^I x - \frac{k}{2}x^2.$$

One can verify that  $E(\tau_g^S(x)) = E(\tau_r^S(x))$ . Therefore

$$\begin{aligned} \text{Var}(\tau_g^S(x)) - \text{Var}(\tau_r^S(x)) &= E(\tau_g^S(x))^2 - E(\tau_r^S(x))^2 \\ &= E\left(\frac{k^2}{4}x^4\right) - (p^I)^2 E(x^2) + p^I E(kx^3) - E\left(\frac{k^2}{4}x^4\right) \\ &= p^I \left( E(k^3x) - (p^I)^2 E(x^2) \right) = \left( p^I E(k^3x) - \frac{E(k^2x^2)}{E(x)} \right) E(x) \\ &= \frac{kp^I}{E(x)} \left( E(x^3)E(x) - E(x^2)^2 \right) \geq 0. \end{aligned}$$

The last step is by Cauchy's inequality.<sup>7</sup>

## V. Discussions

### Fixed Retail Pricing

This paper assumes that consumer prices are fixed in advance. Fixed pricing is common in electricity markets, especially for residential users, probably because it is technically simple. A drawback of fixed pricing under demand uncertainty is that consumers have no incentive to respond to spot wholesale price (or marginal production cost), which means that the consumption is typically Pareto suboptimal. For example, when the demand is high and thus the marginal production cost is high, it is socially desirable for consumers to cut usage. But under fixed pricing they do not have the incentive to do so.

<sup>7</sup> A little more generally, we can show that the same result holds as long as  $C'''(x) \geq 0$ . Details please see Appendix.

The model of this paper can be reinterpreted to adapt to the cases with variable prices, as long as we can still assume perfectly inelastic ex post demand. From the perspective of producers, the case of vertical separation is equivalent to the case where producers directly sell to final consumers via variable prices, which equal the marginal production costs. In this case, the role of retailers is trivial. Hence the current model can also be viewed as discussing the difference between fixed pricing (vertical integration) and variable pricing (vertical separation). It suggests that fixed pricing and variable pricing result in the same expected profits. However, under variable pricing, producers and consumers tend to take more risk.

### **Electricity industry**

This paper offers some insights on the deregulation of electricity industry. In most countries electricity industry used to be operated by vertically integrated monopolists, which are subject to government regulation. Led by Chile in 1982 and the United Kingdom in 1990, many countries are reforming their electricity industry in order to improve performances. In the reforms, transmission and distribution networks are usually split from the traditional power companies and are still subject to government regulations. Competition is introduced and encouraged at generation and retail stages. Generators and retailers trade in spot wholesale markets.

The performances of the reforms are mixed. Some reforms are viewed successful, *e.g.*, the Nordic market (including Denmark, Finland, Norway and Sweden). Amundsen and Bergman (2006) suggest that the successful reform in the Nordic market be attributed to a simple but sound market design, successful dilution of market power, strong political support

of deregulation, and high proportion of hydroelectric energy.<sup>8</sup> Other reforms might be less satisfactory, *e.g.*, the British market (Green and Newbery 1992) and Californian market (Borenstein, Bushnell and Wolak, 2002; Joskow and Kahn, 2002) before 2001. Green and Newbery (1992) find that the competition in supply schedules in the British electricity spot market implies a high markup on marginal cost and substantial deadweight losses. Wolak (2003) “diagnoses” the Californian electricity crisis during 2000-2001. He emphasizes the role of supplier market power in causing the crisis, and suggests that the Federal Energy Regulatory Commission (FERC) should regulate, rather than simply monitor, wholesale electricity markets.

While studies in the literature emphasize the role of market power, the present paper underlines the role of vertical industrial structure in influencing the industrial performances. The key assumptions invoked include aggregate demand uncertainty and increasing marginal generation costs. It suggests that at least under perfect competition, vertical separation of generation and retail does not affect the expected profits of the firms. However, it tends to enhance the risk faced by the firms. In particular, the market tends to be less stable because some of the firms, especially retailers, might not be able to survive large demand shocks. We suggest that governments should encourage generators to serve as “load serving entities (LSEs)” directly.<sup>9</sup>

## VI. Concluding Remarks

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<sup>8</sup> It should be noted that the assessment is not based on a quantitative assessment of the market outcomes before versus after the reform.

<sup>9</sup> As suggested by Wolak (2003), market power could lead to the failure of spot wholesale markets. Vertical integration might also be a solution to the problems. Details are left for future studies.

This paper studies the relationship between vertical market structure and risk faced by firms. In the model, the aggregate demand is stochastic, marginal production cost is increasing, and average retail cost is constant. It is also assumed that firms are price-takers and risk-neutral and retail prices must be determined before the demand realizes. It is shown that the vertical market structure does not influence the expected profits for firms or the equilibrium prices for consumers. However, it influences the risk faced by both producers and retailers. Specifically, the upstream and downstream profits under vertical separation are negatively correlated with each other. Hence the separation exaggerates the risk faced by the firms. It is also likely for the upstream firms to take more risk under separation than under integration. Hence the market tends to be less stable under vertical separation.

The findings of this paper have clear policy implications. We suggest that splitting traditional power firms into independent generators and wholesalers results in more financial risk for firms. The approach makes the whole system less stable. Generators should be encouraged to sell directly to final consumers, or hold significant stakes retail firms.

The analysis of this paper can also be interpreted as comparing fixed pricing (which resembles that of vertical integration) and variable pricing (which resembles that of vertical separation). It suggests that fixed pricing reduce the risk faced by firms. It might also benefit buyers if they dislike large fluctuations in their electricity bills.

We conjecture that in the model considered in the paper, the equilibrium final price should be higher under vertical separation if firms are risk-averse. Further studies are needed to verify this conjecture.

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### Appendix

We will show that the variance of producer profit is larger under vertical separation as long as the cost function satisfies  $C'''(x) \geq 0$ .

$$\begin{aligned} E(\tau_g^s(x))^2 - E(\tau^l(x))^2 &= E(MC(x)x - C(x))^2 - E(p^l x - C(x))^2 \\ &= E[(MC(x)x + p^l x - 2C(x))(MC(x) - p^l)x] \\ &= E[H(x)(MC(x) - p^l)x], \end{aligned}$$

where  $H(x) \equiv MC(x)x + p^l x - 2C(x)$ . Since  $MC(q^l) = p^l$ ,

$$(MC(x) - p^l)x \begin{cases} > 0 & \text{for } x > q^l \\ < 0 & \text{for } x < q^l. \end{cases}$$

Since

$$H'(x) = MC'(x)x + p^l - MC(x),$$

we have

$$H'(x) = MC'(x)x + p^l - MC(x) > 0$$

for  $x < q^l$ . If  $C'''(x) \geq 0$ , we have  $H'(x) > 0$  for  $x > q^l$ . Hence

$$\begin{aligned}
E\left(\tau_g^s(x)\right)^2 - E\left(\tau^l(x)\right)^2 &= E\left[H(x)(MC(x) - p^l)x\right] \\
&> E\left[H(q')(MC(x) - p^l)xI_{(x < q')} + H(q')(MC(x) - p^l)xI_{(x < q')}\right] \\
&= E\left[H(q')(MC(x) - p^l)x\right] = 0,
\end{aligned}$$

where

$$I_{(x < q')} \begin{cases} = 1 & \text{for } x < q' \\ = 0 & \text{for } x > q' \end{cases} \quad \text{and} \quad I_{(x > q')} \begin{cases} = 1 & \text{for } x > q' \\ = 0 & \text{for } x < q' \end{cases}$$